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# توزیع فشار در سیال ساکن

مقدمه

# (۱) توزیع فشار در اتمسفر و دریاها،

(۲) نیروهای مؤثر بر سطوح غوطه‌ور تخت و خمیده، (۳) نیروی شناوری، (۴) اجسام شناور، (۵) توزیع فشار در مایع داخل یک ظرف شتاب‌دار، (۶) ابزار اندازه‌گیری فشار سیالات.

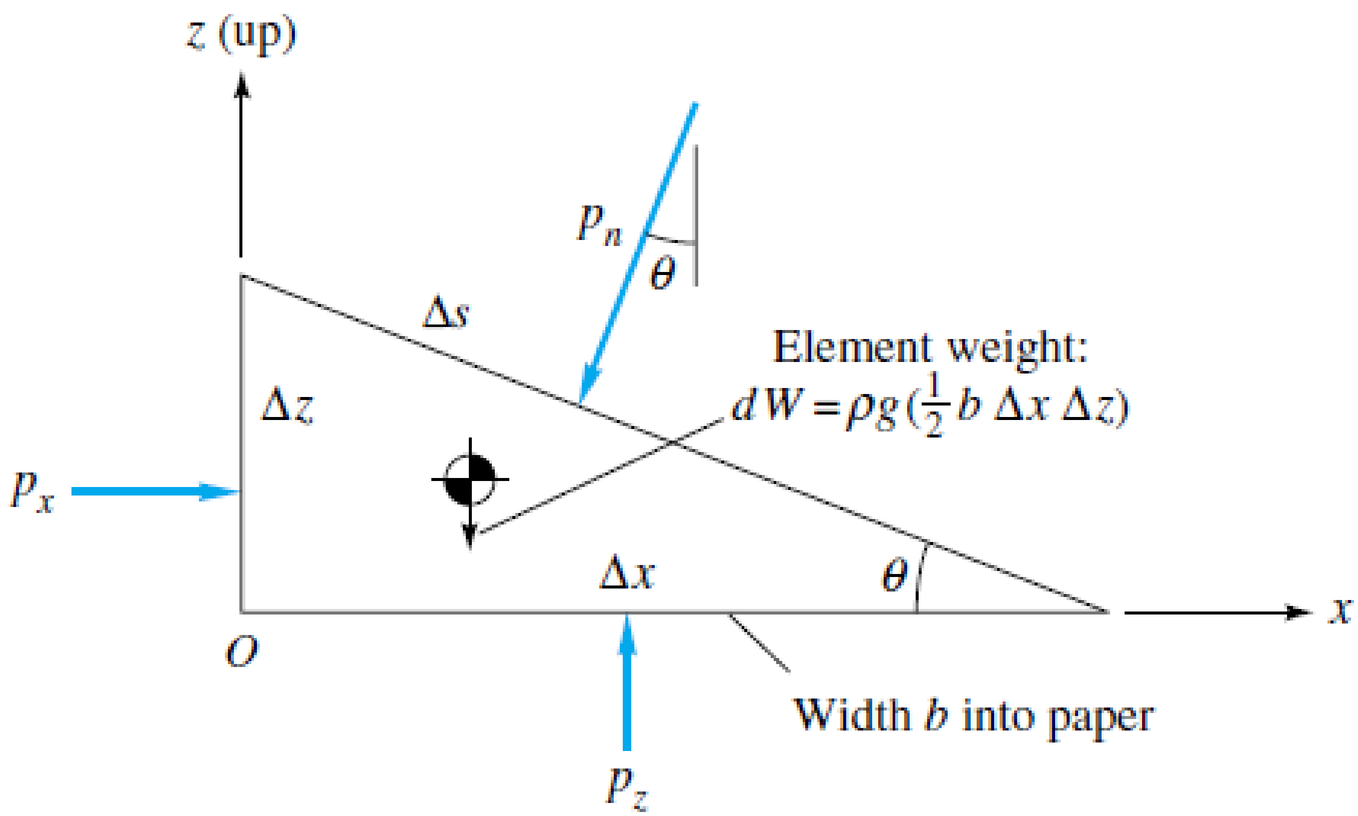
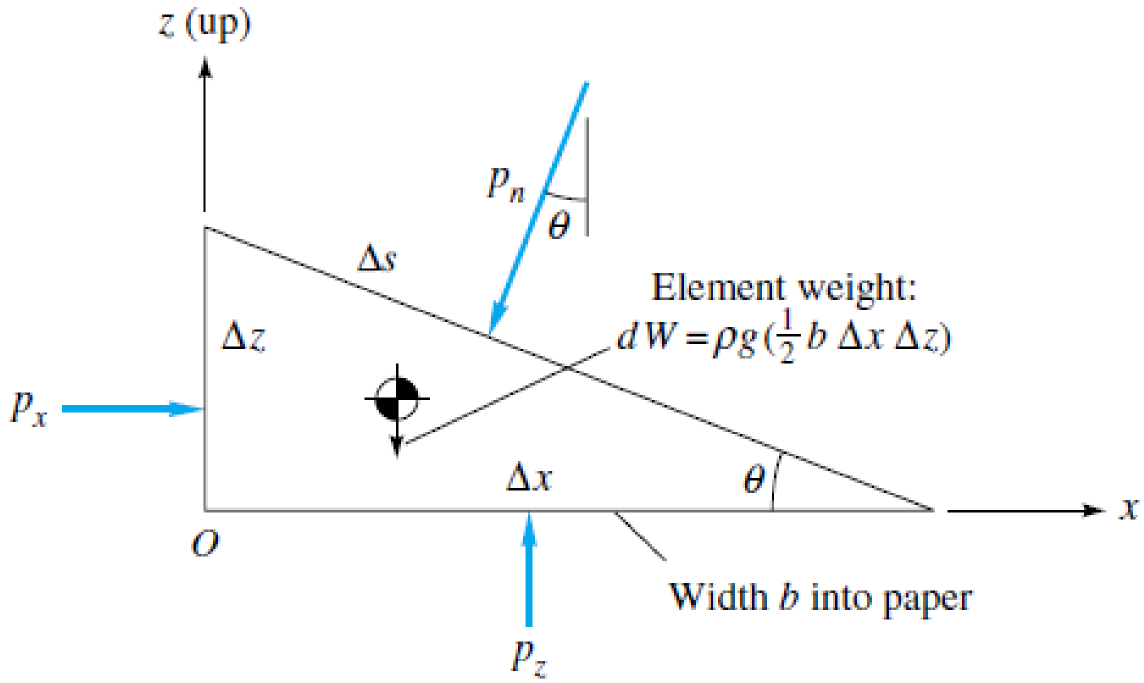
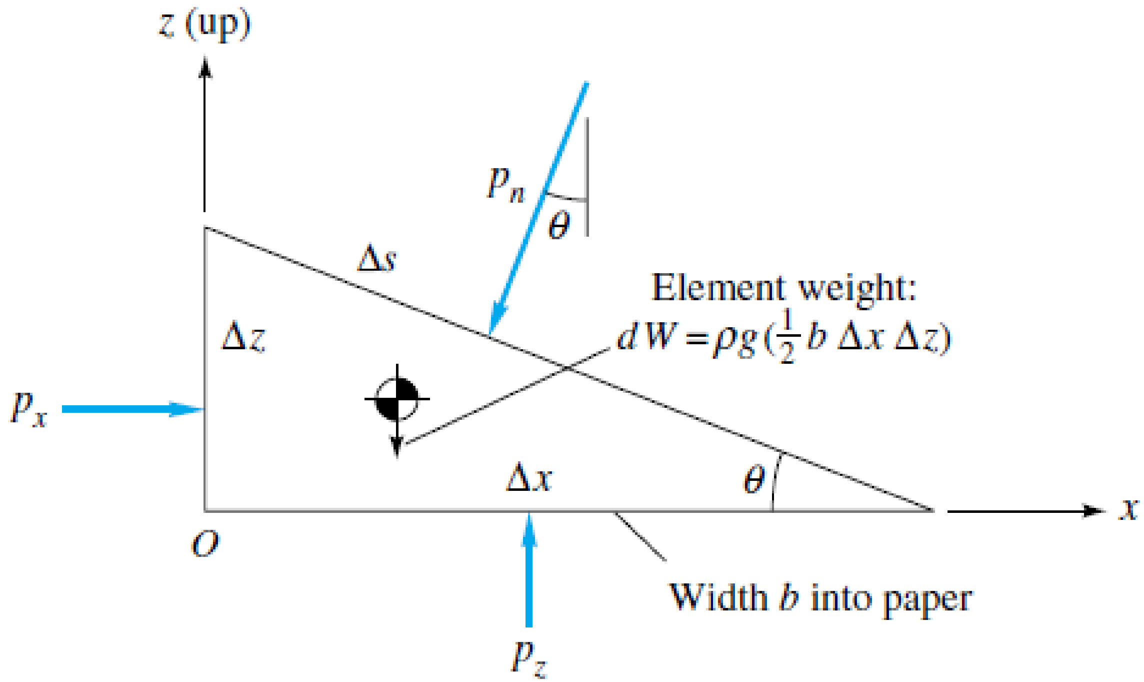



**Fig. 2.1** Equilibrium of a small wedge of fluid at rest.




$$\sum F_x = 0 = p_x b\,\Delta z - p_n b\,\Delta s \sin\theta$$

$$\sum F_z = 0 = p_z b\,\Delta x - p_n b\,\Delta s \cos\theta - \tfrac{1}{2}\gamma b\,\Delta x\,\Delta z$$

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but the geometry of the wedge is such that

$$\Delta s \sin\theta = \Delta z \qquad \Delta s \cos\theta = \Delta x$$

Substitution into Eq. (2.1) and rearrangement give

$$p_x = p_n \qquad p_z = p_n + \tfrac{1}{2}\gamma\, \Delta z$$

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These relations illustrate two important principles of the hydrostatic, or shear-free, condition: (1) There is no pressure change in the horizontal direction, and (2) there is a vertical change in pressure proportional to the density, gravity, and depth change. We shall exploit these results to the fullest, starting in Sec. 2.3.

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In the limit as the fluid wedge shrinks to a "point," $\Delta z \to 0$ and Eqs. (2.3) become

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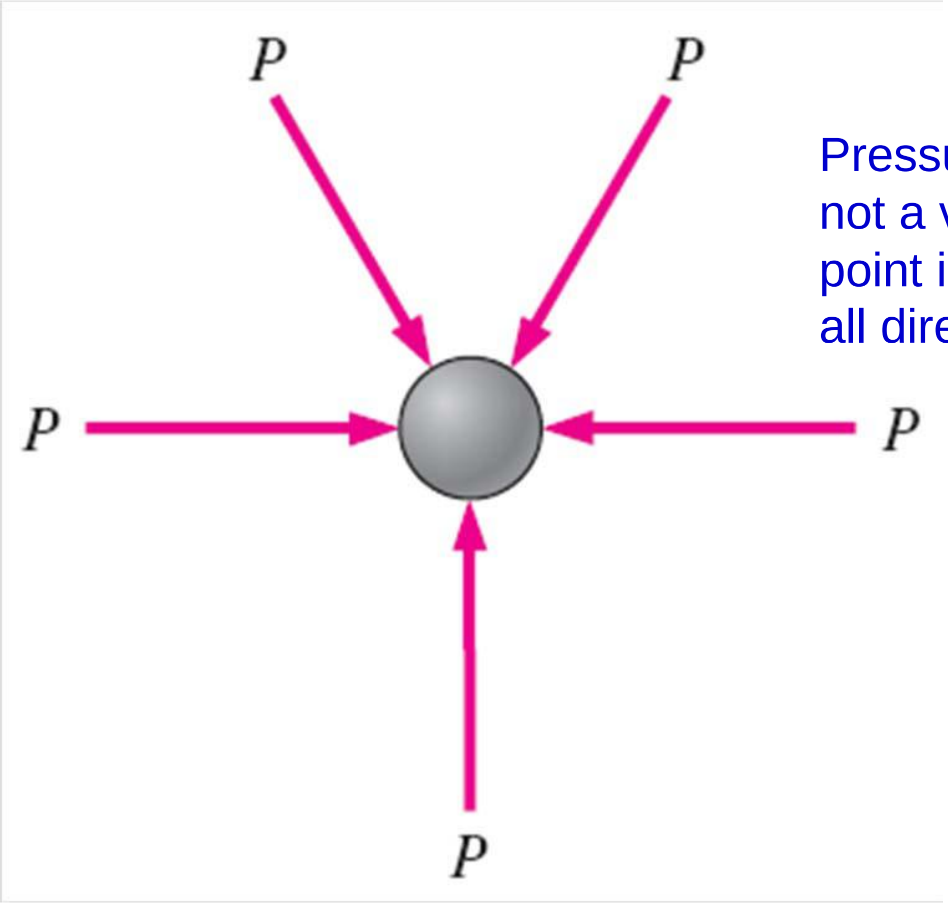


Pressure is a *scalar* quantity, not a vector; the pressure at a point in a fluid is the same in all directions.

What about the pressure at a point in a moving fluid? If there are strain rates in a moving fluid, there will be viscous stresses, both shear and normal in general (Sec. 4.3). In that case (Chap. 4) the pressure is defined as the average of the three normal stresses $\sigma_{ii}$ on the element

$$p = -\tfrac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \tag{2.5}$$

The minus sign occurs because a compression stress is considered to be negative whereas $p$ is positive. Equation (2.5) is subtle and rarely needed since the great majority of viscous flows have negligible viscous normal stresses (Chap. 4).

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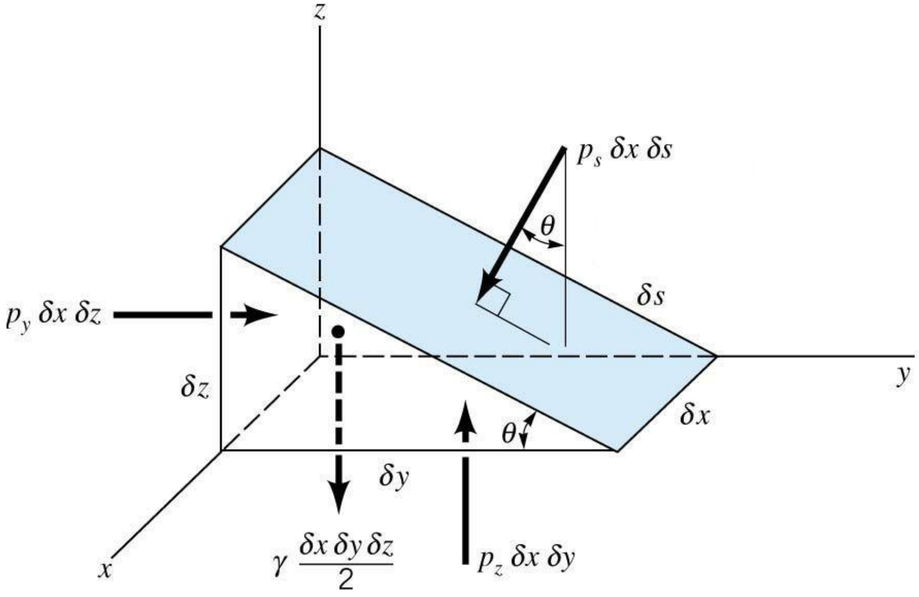
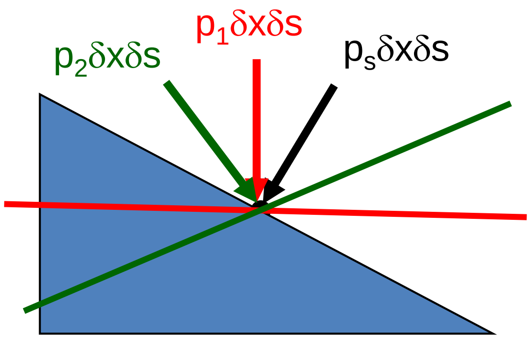
**Pascal's Law: the pressure at a point in a fluid at rest, or in motion, is independent of the direction as long as there are no shearing stresses present.**

# Pressure at a Point: Pascal's Law




$$\boxed{p_s = p_1 = p_2}$$

Note: In dynamic system subject to shear, the normal stress representing the pressure in the fluid is not necessarily the same in all directions. In such a case the pressure is taken as the average of the three directions.

What about the pressure at a point in a moving fluid? If there are strain rates in a moving fluid, there will be viscous stresses, both shear and normal in general (Sec. 4.3). In that case (Chap. 4) the pressure is defined as the average of the three normal stresses $\sigma_{ii}$ on the element

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## Pressure Force on a Fluid Element

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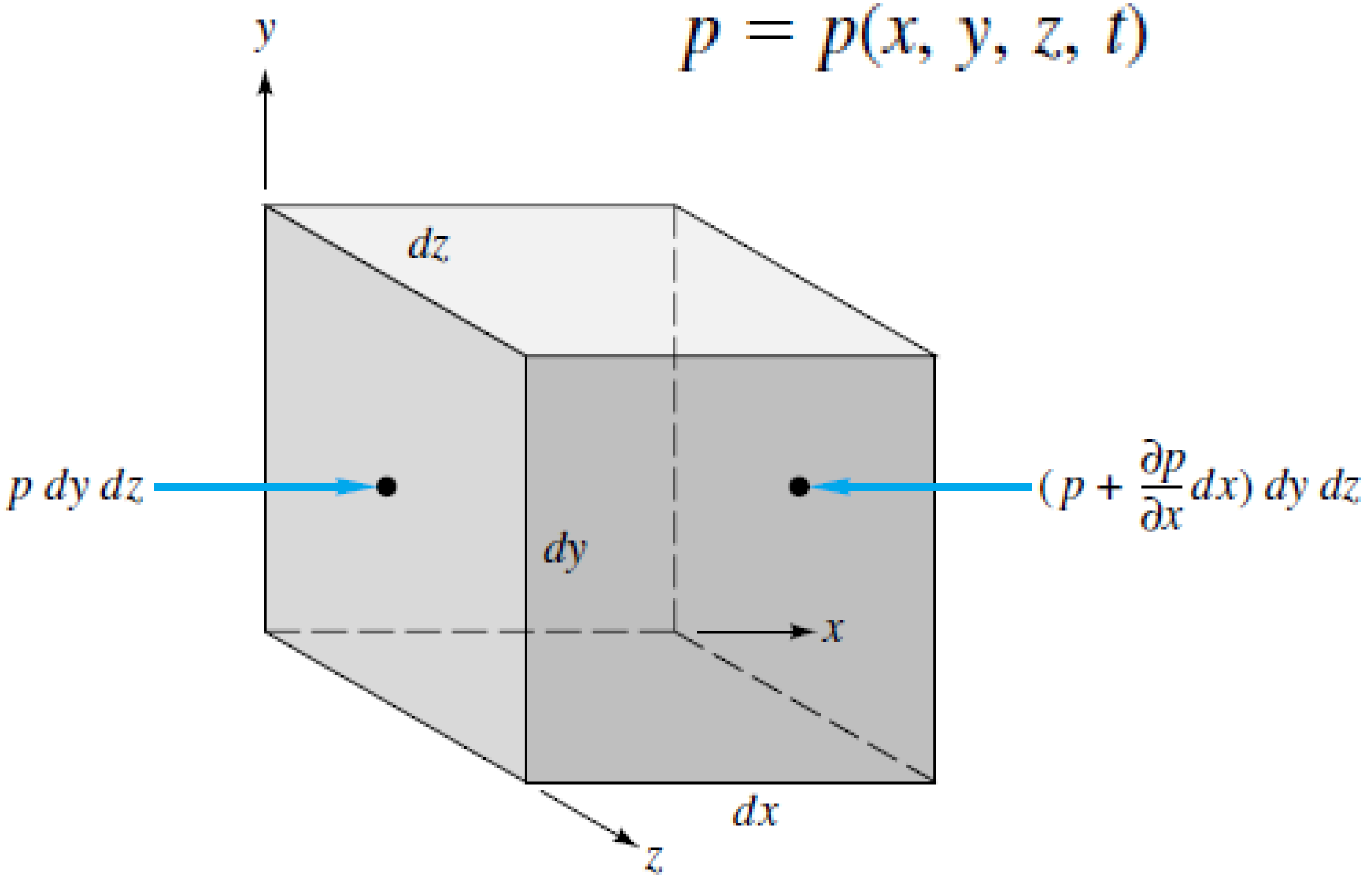


**Fig. 2.2** Net $x$ force on an element due to pressure variation.

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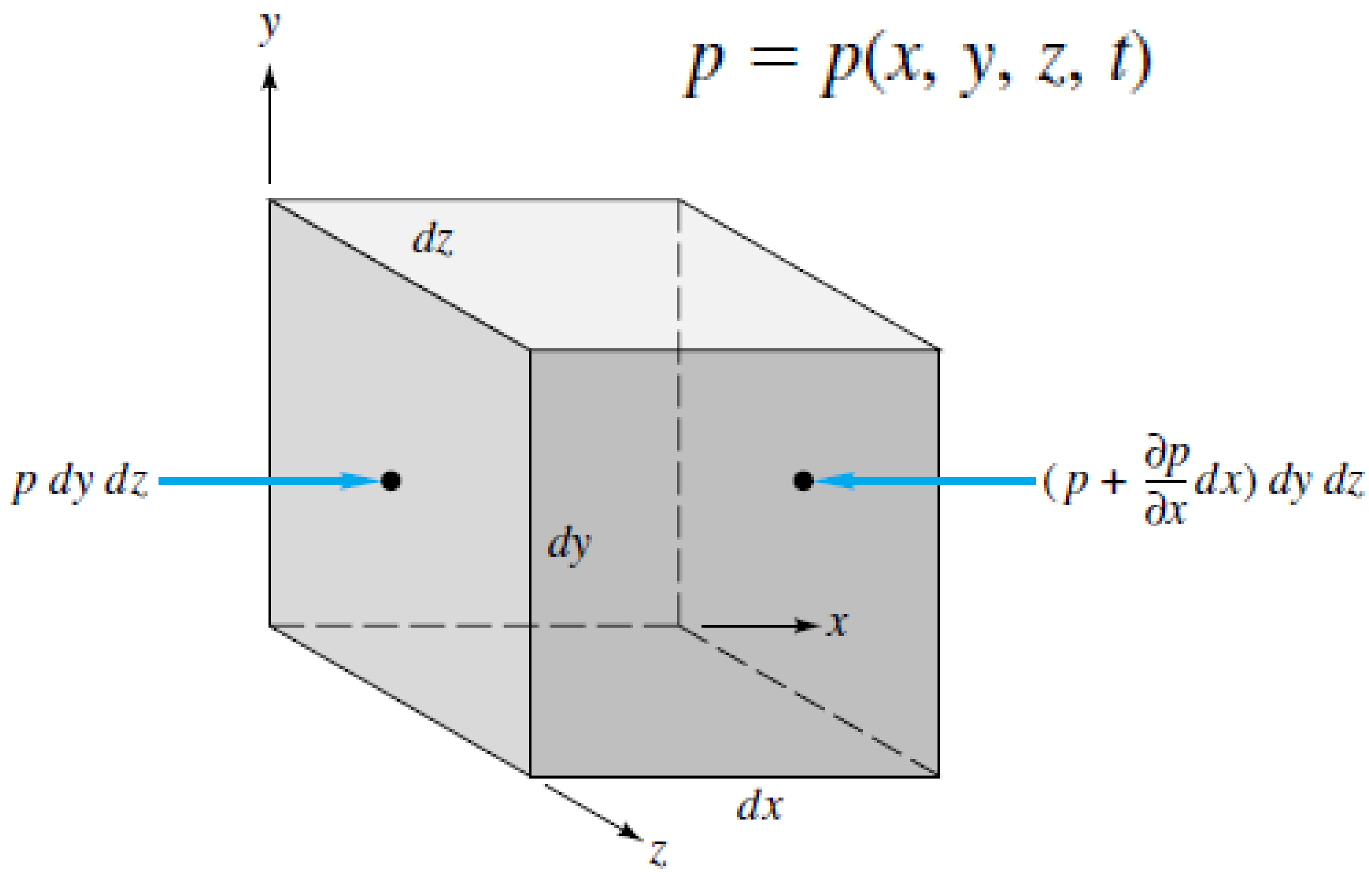


The net force in the $x$ direction on the element in Fig. 2.2 is given by

$$dF_x = p\, dy\, dz - \left(p + \frac{\partial p}{\partial x}\, dx\right) dy\, dz = -\frac{\partial p}{\partial x}\, dx\, dy\, dz$$

In like manner the net force $dF_y$ involves $-\partial p/\partial y$, and the net force $dF_z$ concerns $-\partial p/\partial z$. The total net-force vector on the element due to pressure is

$$d\mathbf{F}_{\text{press}} = \left(-\mathbf{i}\,\frac{\partial p}{\partial x} - \mathbf{j}\,\frac{\partial p}{\partial y} - \mathbf{k}\,\frac{\partial p}{\partial z}\right) dx\,dy\,dz \tag{2.8}$$

We recognize the term in parentheses as the negative vector gradient of $p$. Denoting $\mathbf{f}$ as the net force per unit element volume, we rewrite Eq. (2.8) as

$$\mathbf{f}_{\text{press}} = -\nabla p \tag{2.9}$$

Thus it is not the pressure but the pressure *gradient* causing a net force which must be balanced by gravity or acceleration or some other effect in the fluid.

## 2.2 Equilibrium of a Fluid Element

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In general, there may also be a surface force due to the gradient, if any, of the viscous stresses. For completeness, we write this term here without derivation and consider it more thoroughly in Chap. 4. For an incompressible fluid with constant viscosity, the net viscous force is

$$\mathbf{f}_{\text{VS}} = \mu\left(\frac{\partial^2\mathbf{V}}{\partial x^2} + \frac{\partial^2\mathbf{V}}{\partial y^2} + \frac{\partial^2\mathbf{V}}{\partial z^2}\right) = \mu\nabla^2\mathbf{V} \tag{2.11}$$

The total vector resultant of these three forces—pressure, gravity, and viscous stress—must either keep the element in equilibrium or cause it to move with acceleration **a**. From Newton's law, Eq. (1.2), we have

$$\rho \mathbf{a} = \sum \mathbf{f} = \mathbf{f}_{\text{press}} + \mathbf{f}_{\text{grav}} + \mathbf{f}_{\text{visc}} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{V} \qquad (2.12)$$

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$$\frac{\partial p}{\partial x} = B_x(x, y, z, t) \qquad \frac{\partial p}{\partial y} = B_y(x, y, z, t) \qquad \frac{\partial p}{\partial z} = B_z(x, y, z, t) \tag{2.14}$$

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Examining Eq. (2.13), we can single out at least four special cases:

1. **Flow at rest or at constant velocity:** The acceleration and viscous terms vanish identically, and $p$ depends only upon gravity and density. This is the *hydrostatic* condition. See Sec. 2.3.
2. **Rigid-body translation and rotation:** The viscous term vanishes identically, and $p$ depends only upon the term $\rho(\mathbf{g} - \mathbf{a})$. See Sec. 2.9.
3. **Irrotational motion ($\nabla \times \mathbf{V} \equiv 0$):** The viscous term vanishes identically, and an exact integral called *Bernoulli's equation* can be found for the pressure distribution. See Sec. 4.9.
4. **Arbitrary viscous motion:** Nothing helpful happens, no general rules apply, but still the integration is quite straightforward. See Sec. 6.4.

# Gage Pressure and Vacuum Pressure: Relative Terms

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1. $p > p_a$ *Gage* pressure: $p(\text{gage}) = p - p_a$
2. $p < p_a$ *Vacuum* pressure: $p(\text{vacuum}) = p_a - p$

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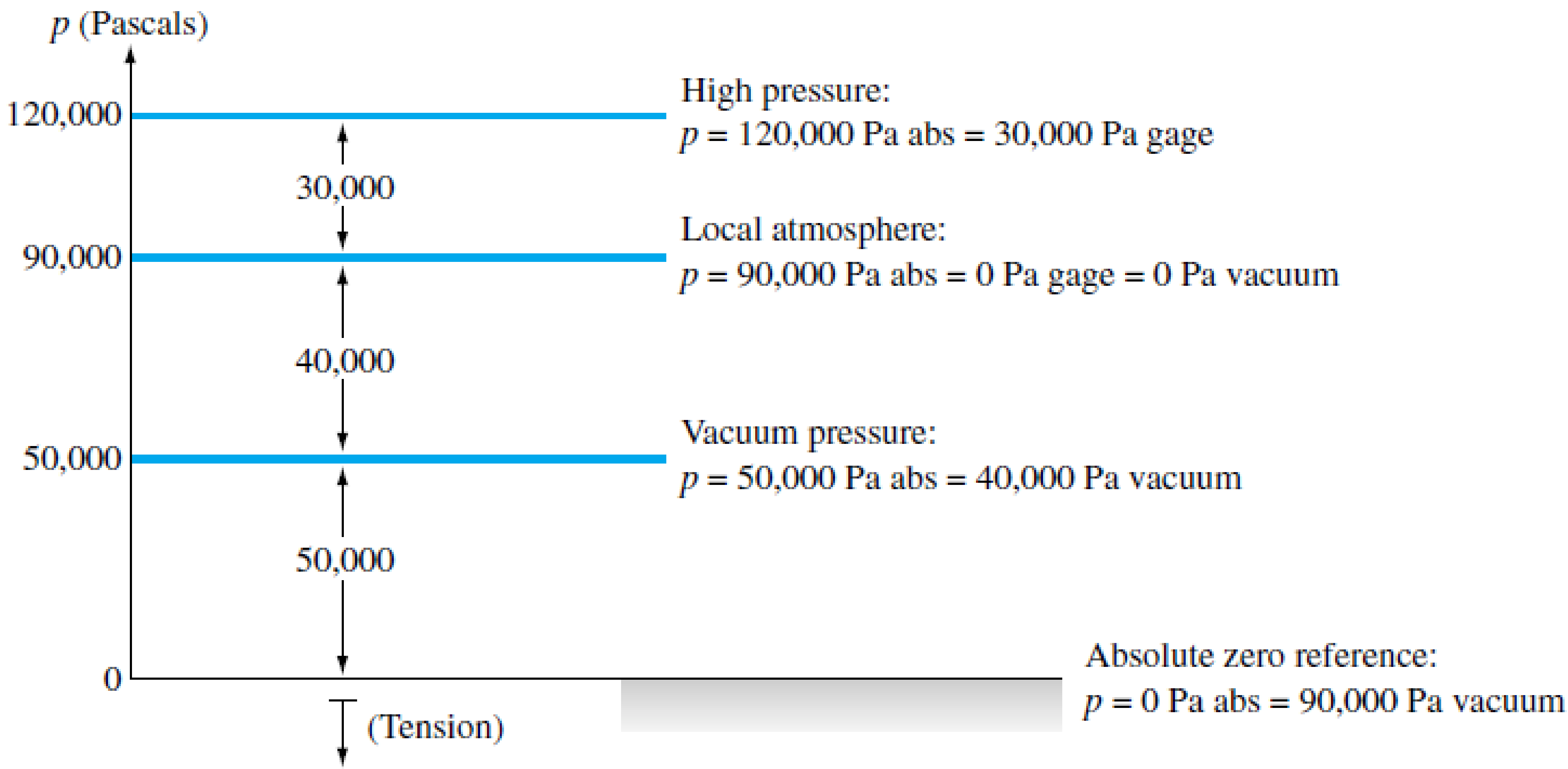

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