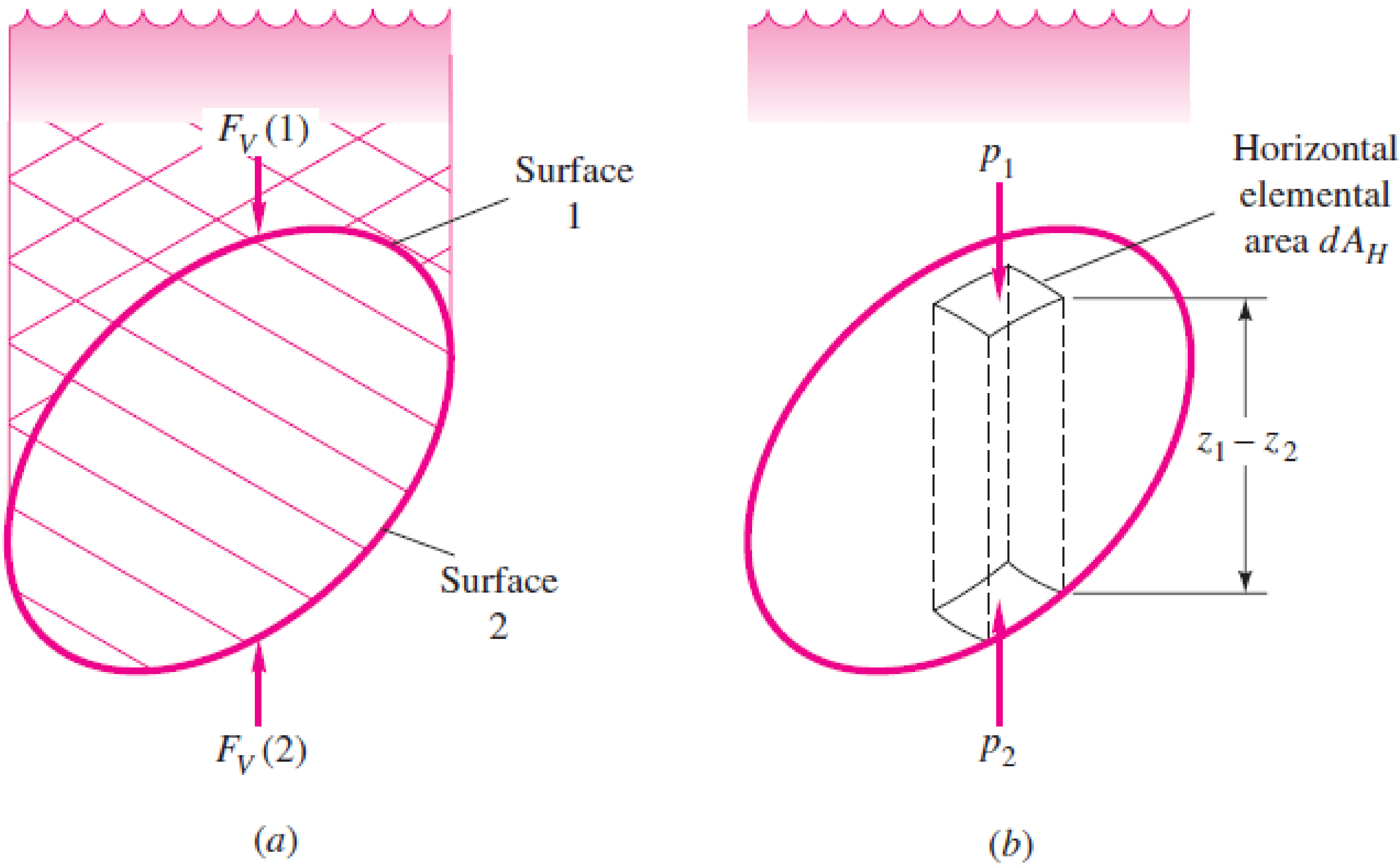
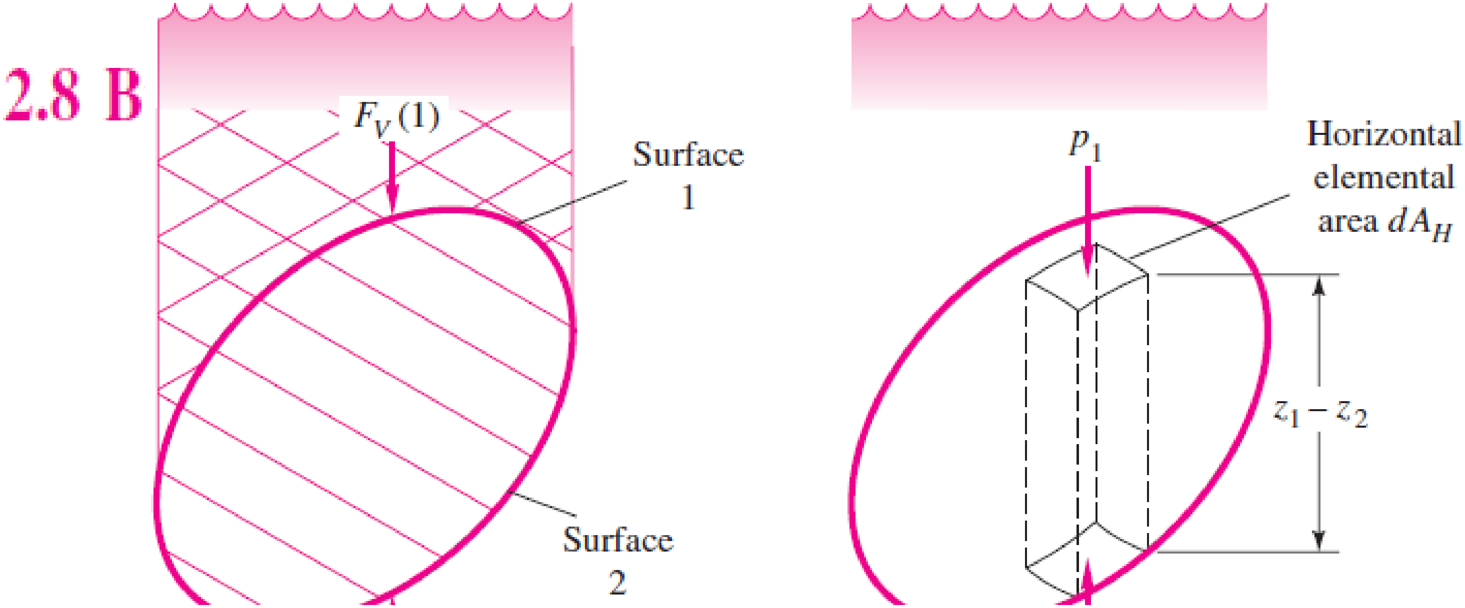
2.8 Buoyancy



$F_V(1)$
Surface 1
Surface 2
$F_V(2)$
(a)
p_1
Horizontal elemental area dA_H
$z_1 - z_2$
p_2
(b)



$$F_B = F_V(2) - F_V(1)$$
$$= \text{(fluid weight above 2)} - \text{(fluid weight above 1)}$$
$$= \text{weight of fluid equivalent to body volume} \quad (2.33)$$

Alternatively, from Fig. 2.16*b*, we can sum the vertical forces on elemental vertical slices through the immersed body:

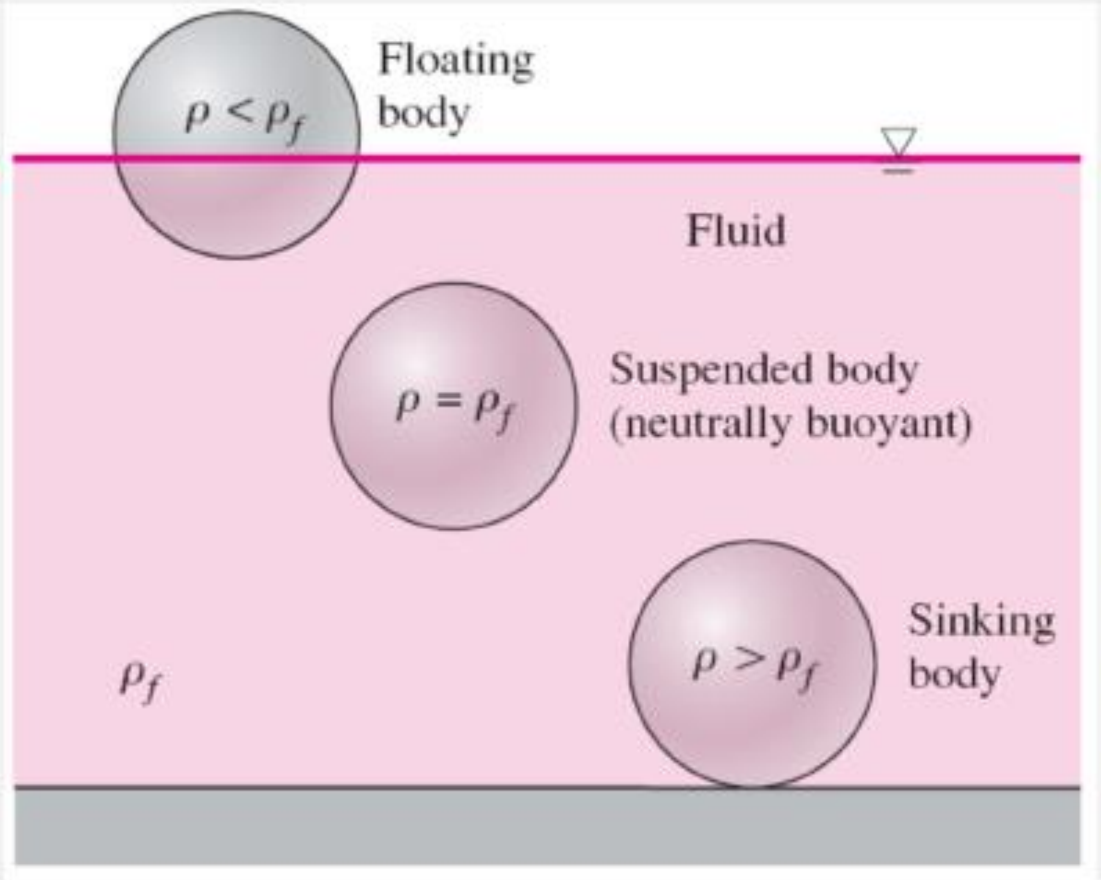
$$F_B = \int_{\text{body}} (p_2 - p_1)\, dA_H = -\gamma \int (z_2 - z_1)\, dA_H = (\gamma)(\text{body volume}) \quad (2.34)$$

These are identical results and equivalent to Archimedes' law 1.

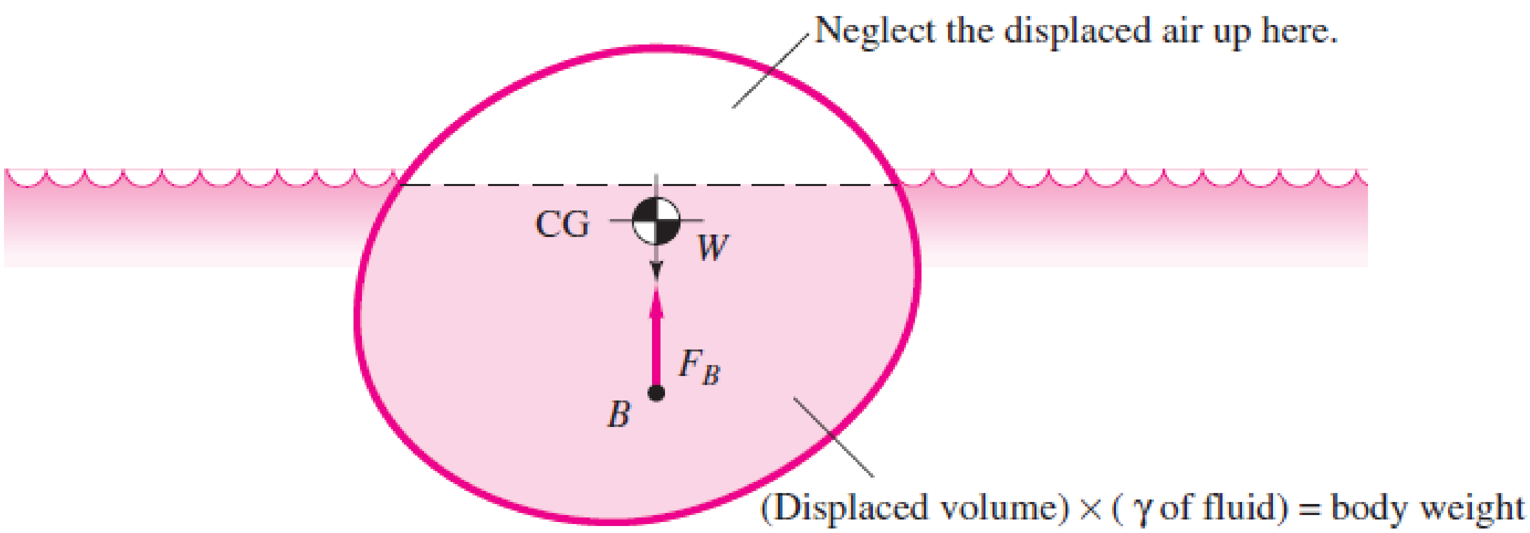
John Ninomiya flying a cluster of 72 helium-filled balloons ov[illegible]ecula, California in April of 2003. The helium balloons displace app[illegible]mately 230 m^3 of air, providing the necessary buoyant force. Don’t try this at home!

For *floating* bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body:

$$F_B = W \rightarrow \rho_f g V_{sub} = \rho_{avg,\,body} g V_{total} \rightarrow \frac{V_{sub}}{V_{total}} = \frac{\rho_{avg,\,body}}{\rho_f}$$



A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid.

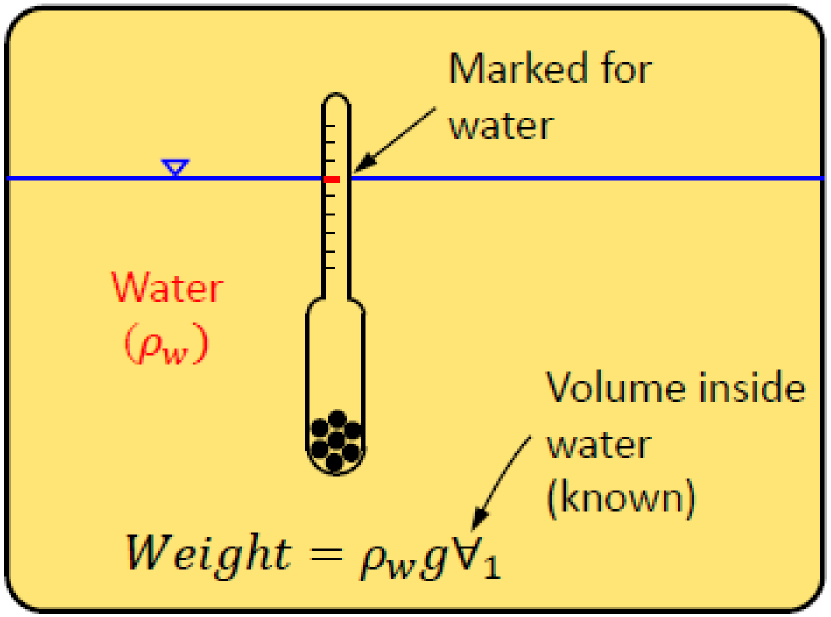


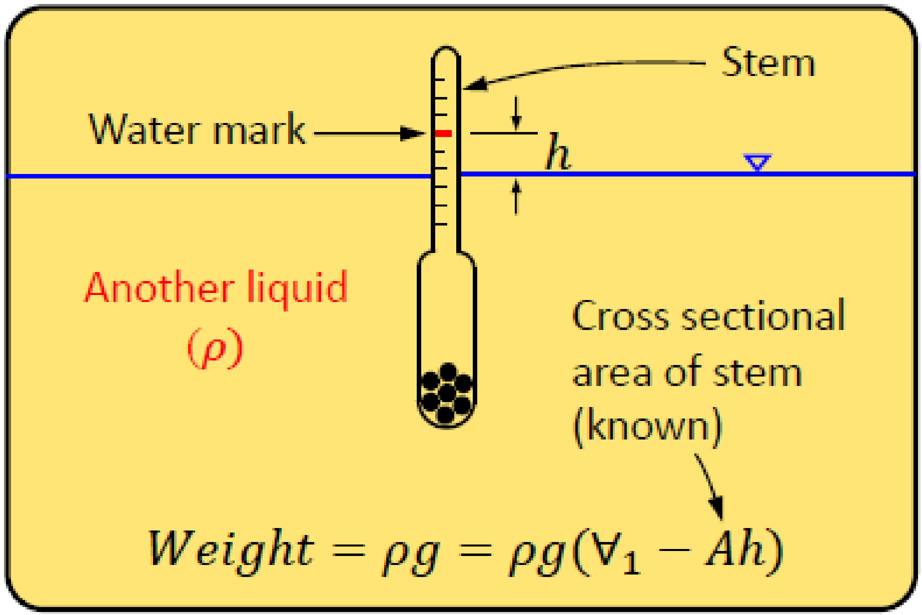
Equation (2.34) can be generalized to a layered fluid (LF) by summing the weights of each layer of density ρ_i displaced by the immersed body:

$$(F_B)_{LF} = \sum \rho_i g(\text{displaced volume})_i \qquad (2.35)$$

Hydrometer

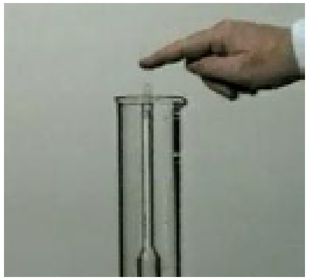
- A hydrometer uses the principle of buoyancy to measure the density of a liquid.
- First it is calibrated by dipping it into a liquid of known density, such as water.





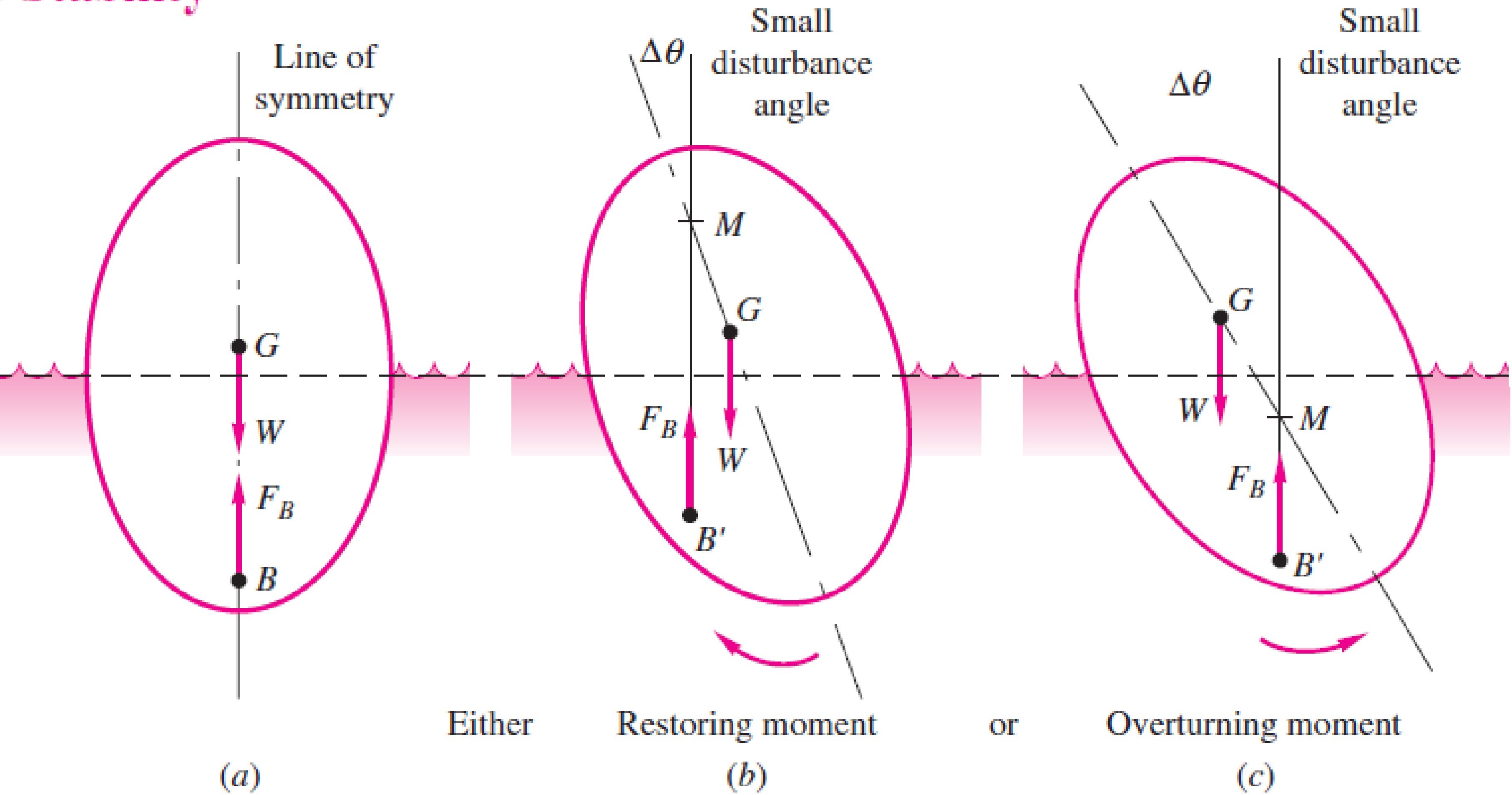
$$\frac{\rho}{\rho_w} = \frac{\forall_1}{\forall_1 - Ah}$$

Movie : Hydrometer



- We read h and then calculate the unknown ρ.
- Stem may be marked so that we can directly read ρ.

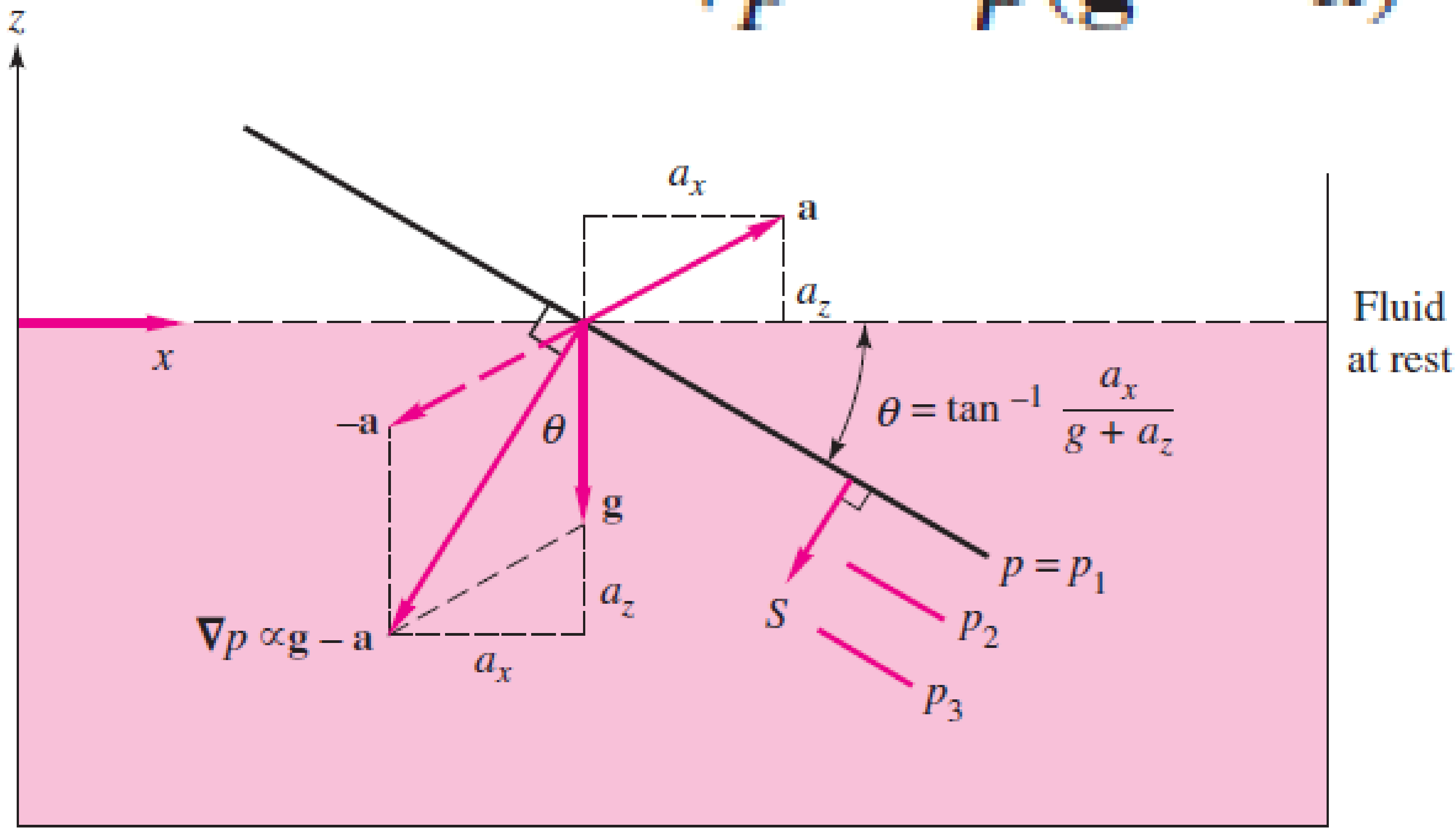
Stability



Line of symmetry
G
W
F_B
B
Δθ
Small disturbance angle
M
G
W
F_B
B′
Δθ
Small disturbance angle
G
W
M
F_B
B′
Either
Restoring moment
or
Overturning moment
(a)
(b)
(c)

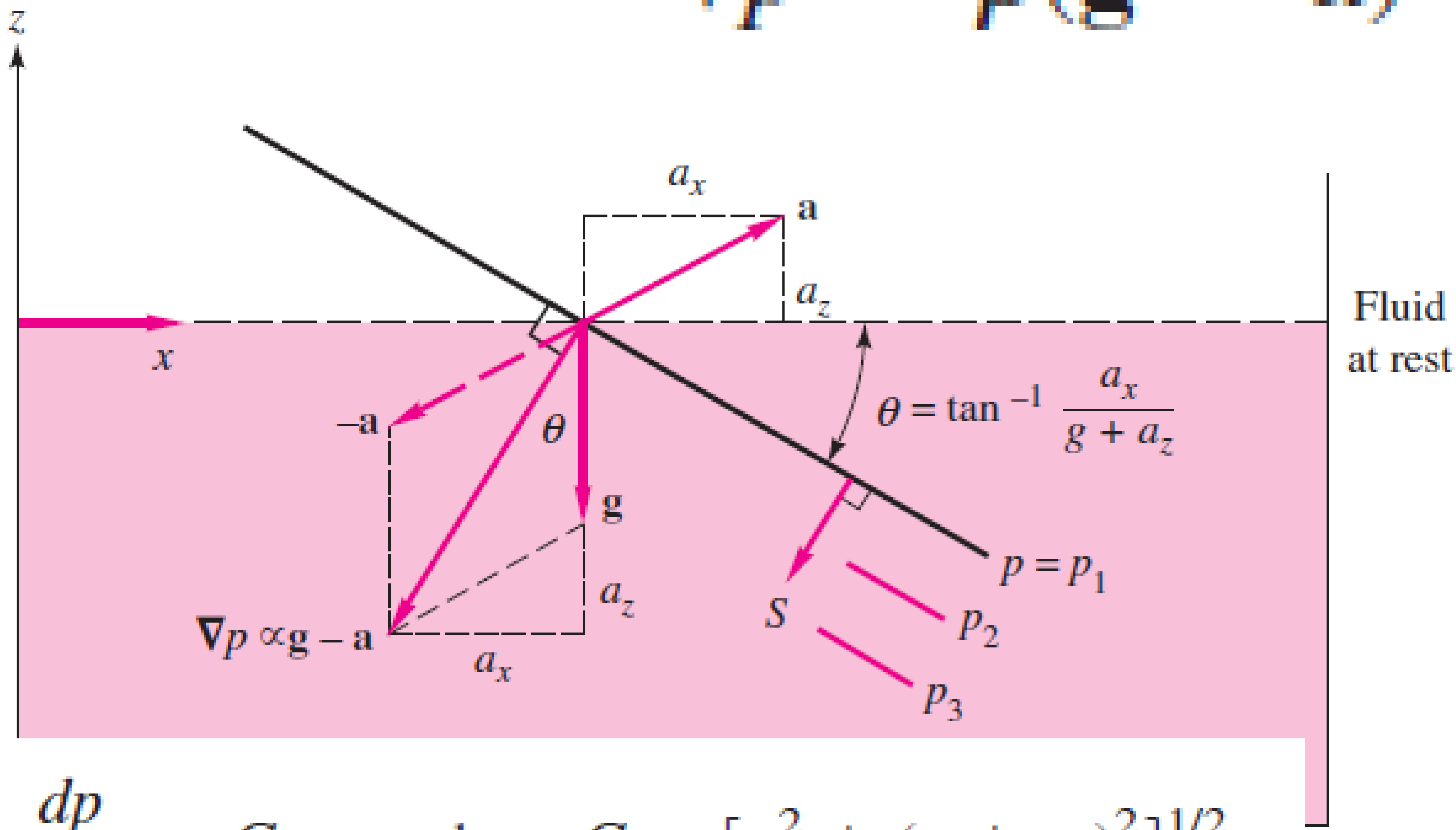
2.9 Pressure Distribution in Rigid-Body Motion

$$\nabla p = \rho(\mathbf{g} - \mathbf{a})$$



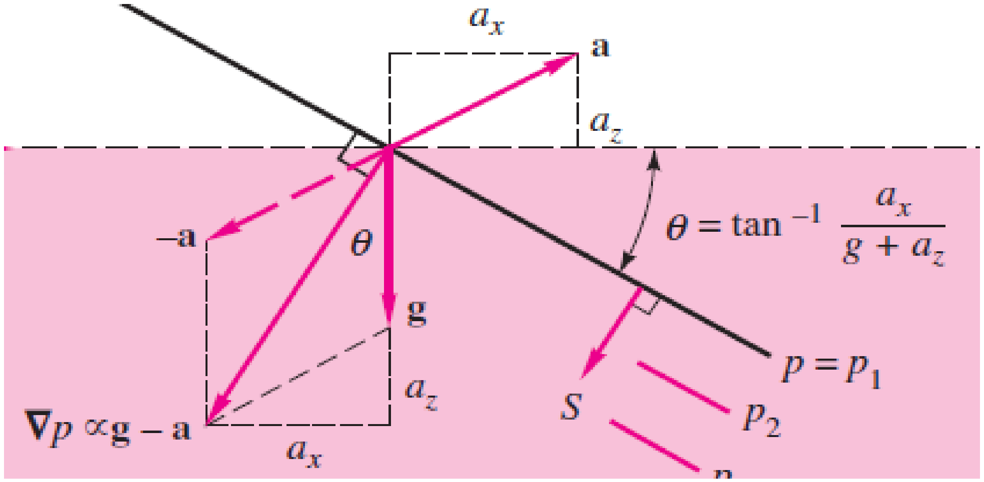
2.9 Pressure Distribution in Rigid-Body Motion

$$\nabla p = \rho(\mathbf{g} - \mathbf{a})$$



$$\frac{dp}{ds} = \rho G \qquad \text{where } G = [a_x^2 + (g + a_z)^2]^{1/2}$$

Acceleration diagram:



For the indicated geometry:

$$\theta = \tan^{-1}\left\{\frac{a_x}{g + a_z}\right\}$$

$$\frac{dP}{ds} = \rho G \qquad \text{where } G = \left\{a_x^2 + (g + a_z)^2\right\}^{1/2}$$

and $\quad P_2 - P_1 = \rho G(s_2 - s_1)$

Note: $P_2 - P_1 \neq \rho\, g(z_2 - z_1)$

and

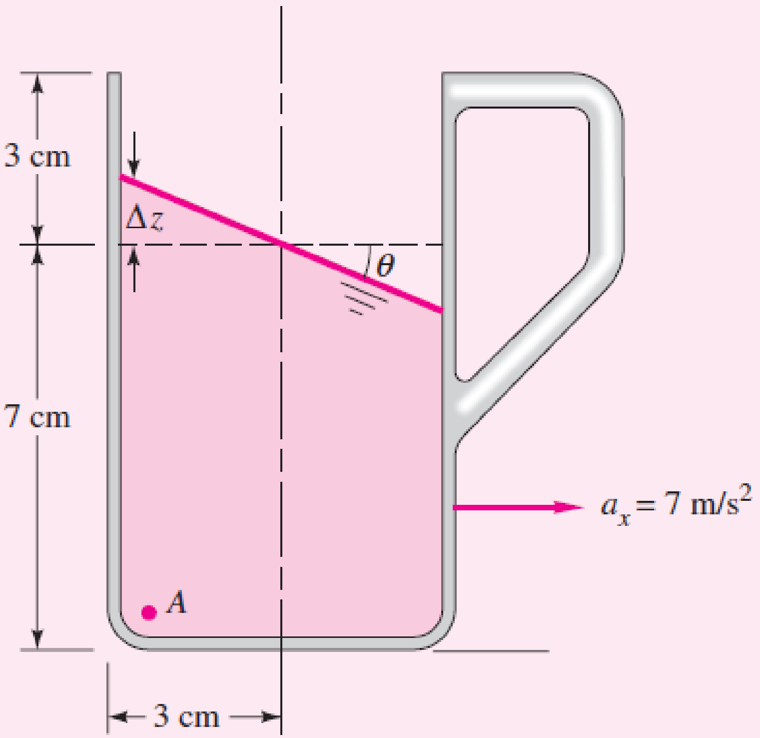
$s_2 - s_1$ is not a vertical dimension

EXAMPLE 2.13

A drag racer rests her coffee mug on a horizontal tray while she accelerates at 7 m/s^2. The mug is 10 cm deep and 6 cm in diameter and contains coffee 7 cm deep at rest. (*a*) Assuming rigid-body acceleration of the coffee, determine whether it will spill out of the mug. (*b*) Calculate the gage pressure in the corner at point *A* if the density of coffee is 1010 kg/m^3.

Solution

- *System sketch:* Figure E2.13 shows the coffee tilted during the acceleration.



E2.13

- *Assumptions:* Rigid-body horizontal acceleration, $a_x = 7\ \text{m/s}^2$. Symmetric coffee cup.
- *Property values:* Density of coffee given as 1010 kg/m^3.
- *Approach (a):* Determine the angle of tilt from the known acceleration, then find the height rise.
- *Solution steps:* From Eq. (2.39), the angle of tilt is given by

$$\theta = \tan^{-1}\frac{a_x}{g} = \tan^{-1}\frac{7.0\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 35.5°$$

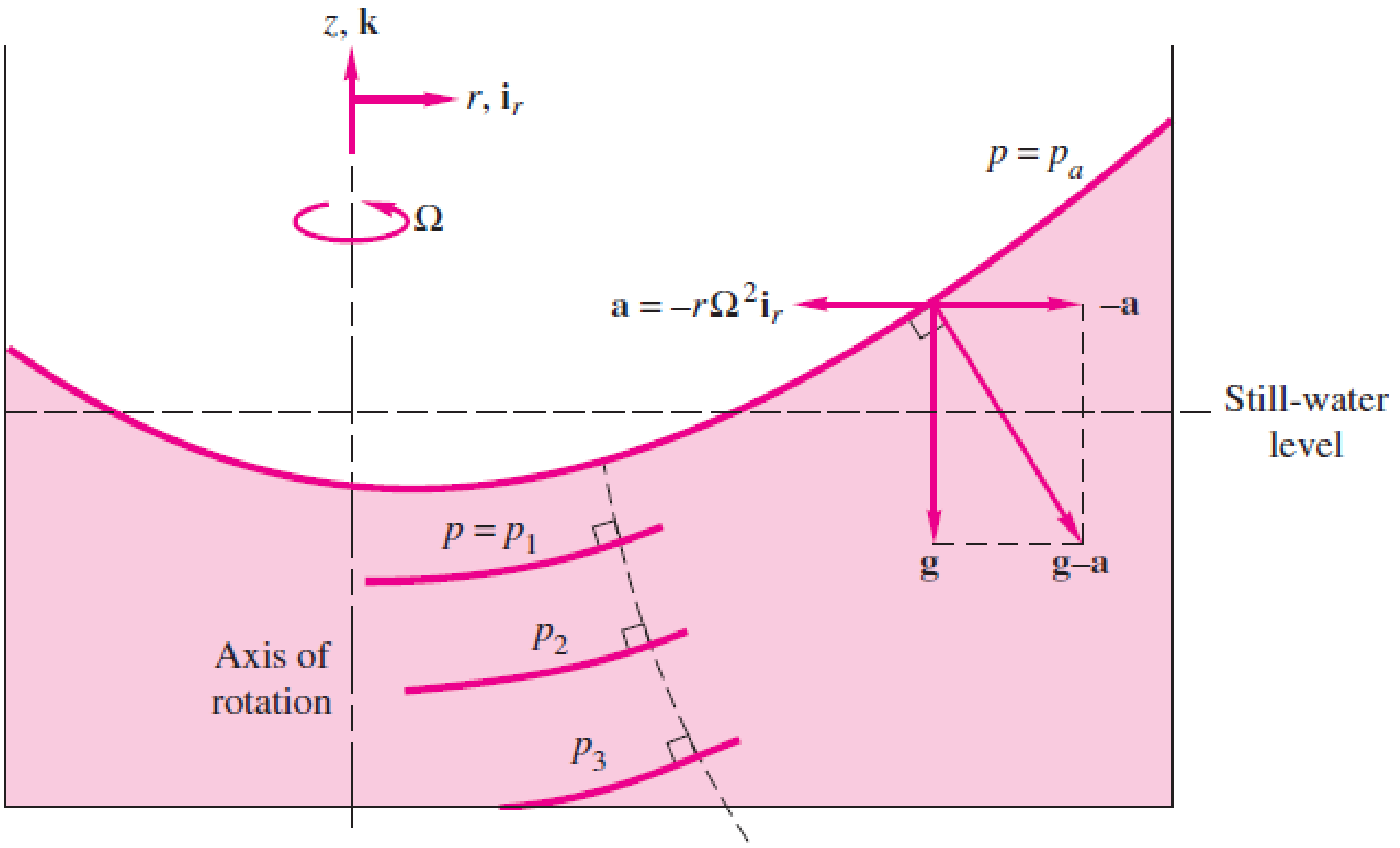
If the mug is symmetric, the tilted surface will pass through the center point of the rest position, as shown in Fig. E2.13. Then the rear side of the coffee free surface will rise an amount Δz given by

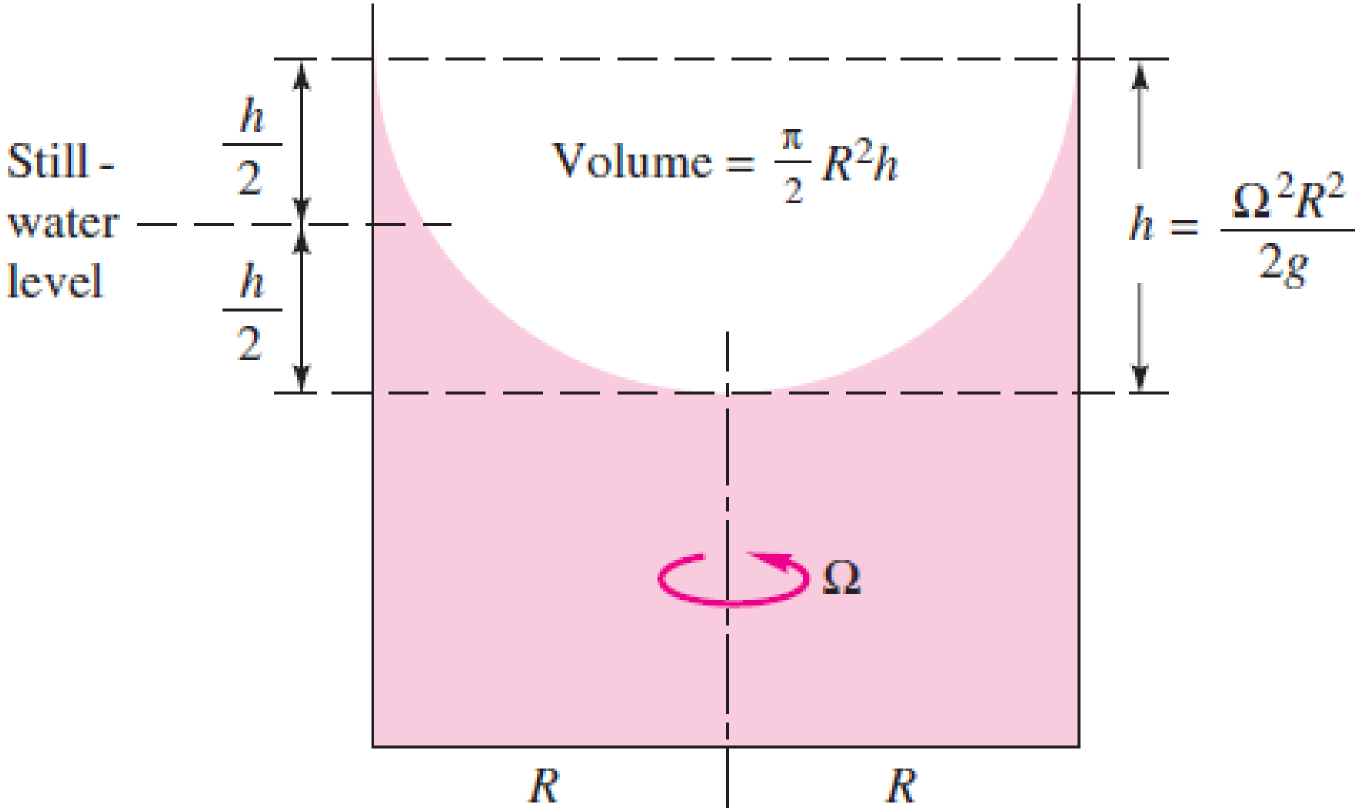
$$\Delta z = (3\ \text{cm})(\tan 35.5°) = 2.14\ \text{cm} < 3\ \text{cm} \qquad \text{therefore no spilling} \qquad \textit{Ans. (a)}$$

- *Comment (a):* This solution neglects sloshing, which might occur if the start-up is uneven.
- *Approach (b):* The pressure at A can be computed from Eq. (2.40), using the perpendicular distance Δs from the surface to A. When at rest, $p_A = \rho g h_{\text{rest}} = (1010\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.07\ \text{m}) = 694$ Pa. When accelerating,

$$p_A = \rho G\,\Delta s = \left(1010\frac{\text{kg}}{\text{m}^3}\right)\left[\sqrt{(9.81)^2 + (7.0)^2}\right][(0.07 + 0.0214)\cos 35.5°] \approx 906\ \text{Pa} \quad \textit{Ans. (b)}$$

Rigid-Body Rotation





$$\boldsymbol{\Omega} = \mathbf{k}\Omega \quad \mathbf{r}_0 = \mathbf{i}_r r \tag{2.41}$$

Then the acceleration is given by

$$\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_0) = -r\Omega^2\mathbf{i}_r \tag{2.42}$$

as marked in the figure, and Eq. (2.38) for the force balance becomes

$$\nabla p = \mathbf{i}_r \frac{\partial p}{\partial r} + \mathbf{k}\frac{\partial p}{\partial z} = \rho(\mathbf{g} - \mathbf{a}) = \rho(-g\mathbf{k} + r\Omega^2\mathbf{i}_r)$$

$$\nabla p = \mathbf{i}_r \frac{\partial p}{\partial r} + \mathbf{k}\frac{\partial p}{\partial z} = \rho(\mathbf{g} - \mathbf{a}) = \rho(-g\mathbf{k} + r\Omega^2\mathbf{i}_r)$$

Equating like components, we find the pressure field by solving two first-order partial differential equations:

$$\frac{\partial p}{\partial r} = \rho r \Omega^2 \qquad \frac{\partial p}{\partial z} = -\gamma \tag{2.43}$$

The right-hand sides of (2.43) are known functions of r and z. One can proceed as follows: Integrate the first equation "partially," holding z constant, with respect to r. The result is

$$p = \tfrac{1}{2}\rho r^2 \Omega^2 + f(z) \tag{2.44}$$

where the "constant" of integration is actually a function $f(z)$.[2] Now differentiate this with respect to z and compare with the second relation of (2.43):

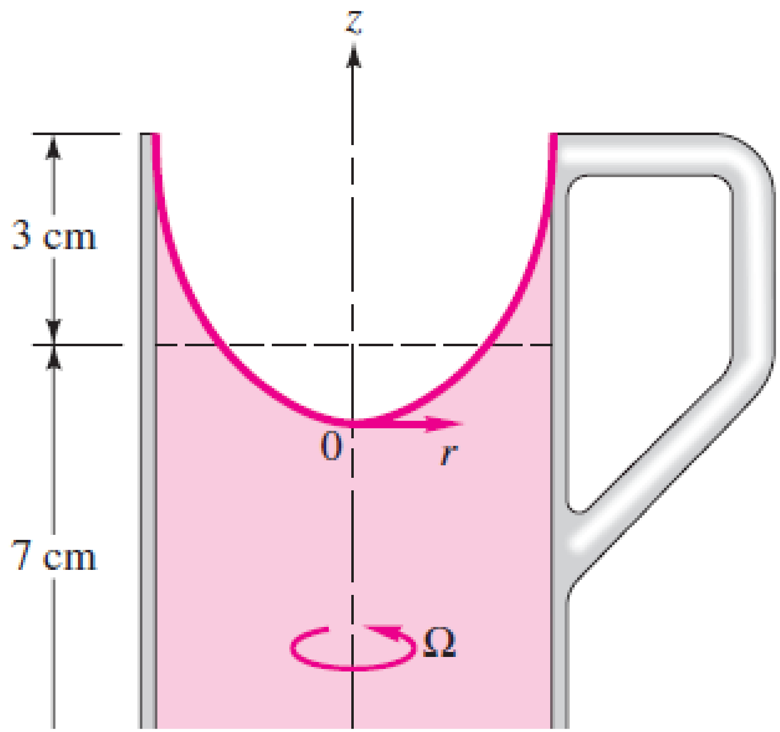
$$\frac{\partial p}{\partial z} = 0 + f'(z) = -\gamma$$

or

$$f(z) = -\gamma z + C$$

where C is a constant. Thus Eq. (2.44) now becomes

$$p = \text{const} - \gamma z + \tfrac{1}{2}\rho r^2 \Omega^2$$



This is the pressure distribution in the fluid. The value of C is found by specifying the pressure at one point. If $p = p_0$ at $(r, z) = (0, 0)$, then $C = p_0$. The final desired distribution is

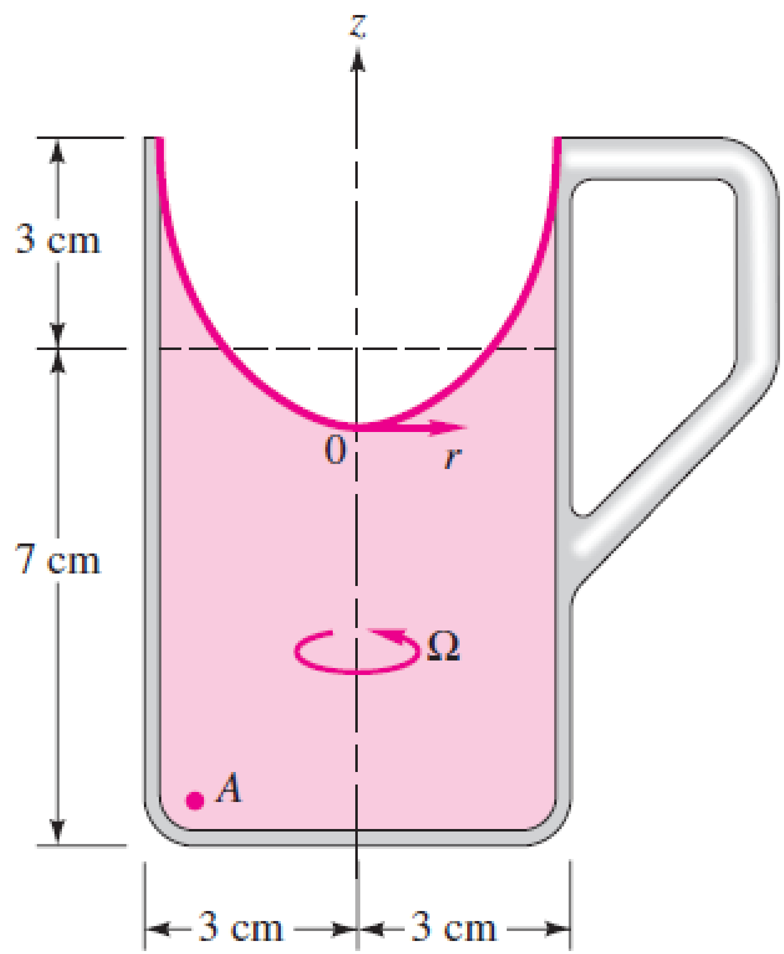
$$p = p_0 - \gamma z + \tfrac{1}{2}\rho r^2 \Omega^2 \tag{2.46}$$

The pressure is linear in z and parabolic in r. If we wish to plot a constant-pressure surface, say, $p = p_1$, Eq. (2.45) becomes

$$z = \frac{p_0 - p_1}{\gamma} + \frac{r^2 \Omega^2}{2g} = a + br^2 \tag{2.47}$$

EXAMPLE 2.14

The coffee cup in Example 2.13 is removed from the drag racer, placed on a turntable, and rotated about its central axis until a rigid-body mode occurs. Find (*a*) the angular velocity that will cause the coffee to just reach the lip of the cup and (*b*) the gage pressure at point *A* for this condition.



Solution

The cup contains 7 cm of coffee. The remaining distance of 3 cm up to the lip must equal the distance $h/2$ in Fig. 2.23. Thus

$$\frac{h}{2} = 0.03 \text{ m} = \frac{\Omega^2 R^2}{4g} = \frac{\Omega^2 (0.03 \text{ m})^2}{4(9.81 \text{ m/s}^2)}$$

Solving, we obtain

$$\Omega^2 = 1308 \qquad \text{or} \qquad \Omega = 36.2 \text{ rad/s} = 345 \text{ r/min} \qquad \textit{Ans. (a)}$$

To compute the pressure, it is convenient to put the origin of coordinates r and z at the bottom of the free-surface depression, as shown in Fig. E2.14. The gage pressure here is $p_0 = 0$, and point A is at $(r, z) = (3 \text{ cm}, -4 \text{ cm})$. Equation (2.46) can then be evaluated:

$$\begin{aligned} p_A &= 0 - (1010 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(-0.04 \text{ m}) \\ &\quad + \tfrac{1}{2}(1010 \text{ kg/m}^3)(0.03 \text{ m})^2(1308 \text{ rad}^2/\text{s}^2) \\ &= 396 \text{ N/m}^2 + 594 \text{ N/m}^2 = 990 \text{ Pa} \qquad \textit{Ans. (b)} \end{aligned}$$

This is about 43 percent greater than the still-water pressure $p_A = 694$ Pa.

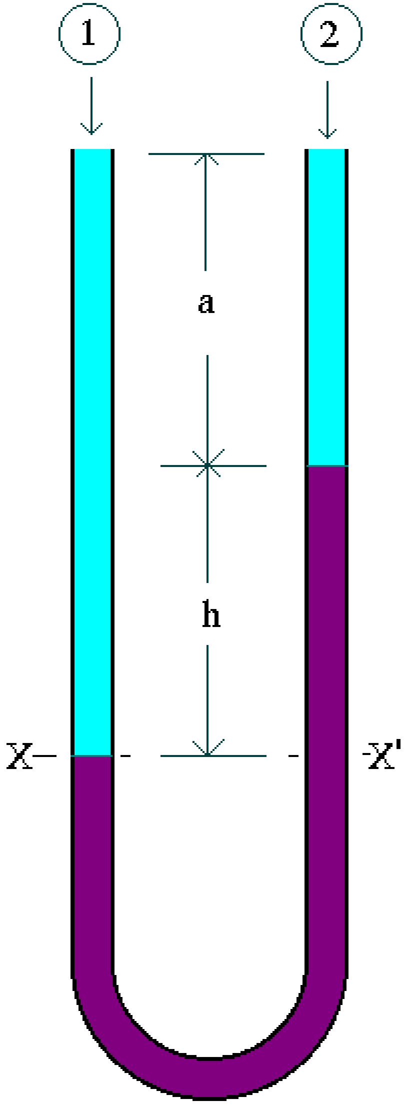
2.10 Pressure Measurement

1. *Gravity-based:* barometer, manometer, deadweight piston.
2. *Elastic deformation:* bourdon tube (metal and quartz), diaphragm, bellows, strain-gage, optical beam displacement.
3. *Gas behavior:* gas compression (McLeod gage), thermal conductance (Pirani gage), molecular impact (Knudsen gage), ionization, thermal conductivity, air piston.
4. *Electric output:* resistance (Bridgman wire gage), diffused strain gage, capacitative, piezoelectric, potentiometric, magnetic inductance, magnetic reluctance, linear variable differential transformer (LVDT), resonant frequency.
5. *Luminescent coatings* for surface pressures [15].

Measurement of Pressure: U-Tube Manometer

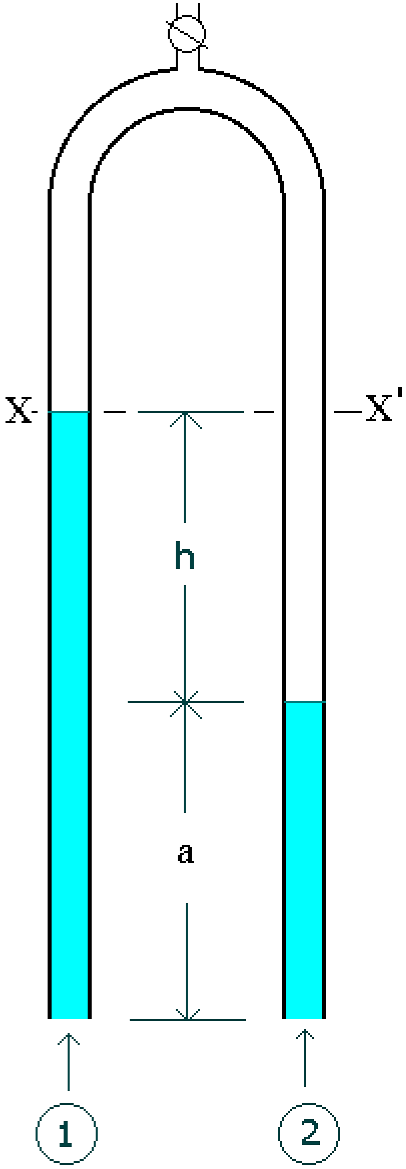
- Simple U - tube manometer
- Inverted U - tube manometer
- U - tube with one leg enlarged
- Two fluid U - tube manometer
- Inclined U - tube manometer

Simple U - tube manometer



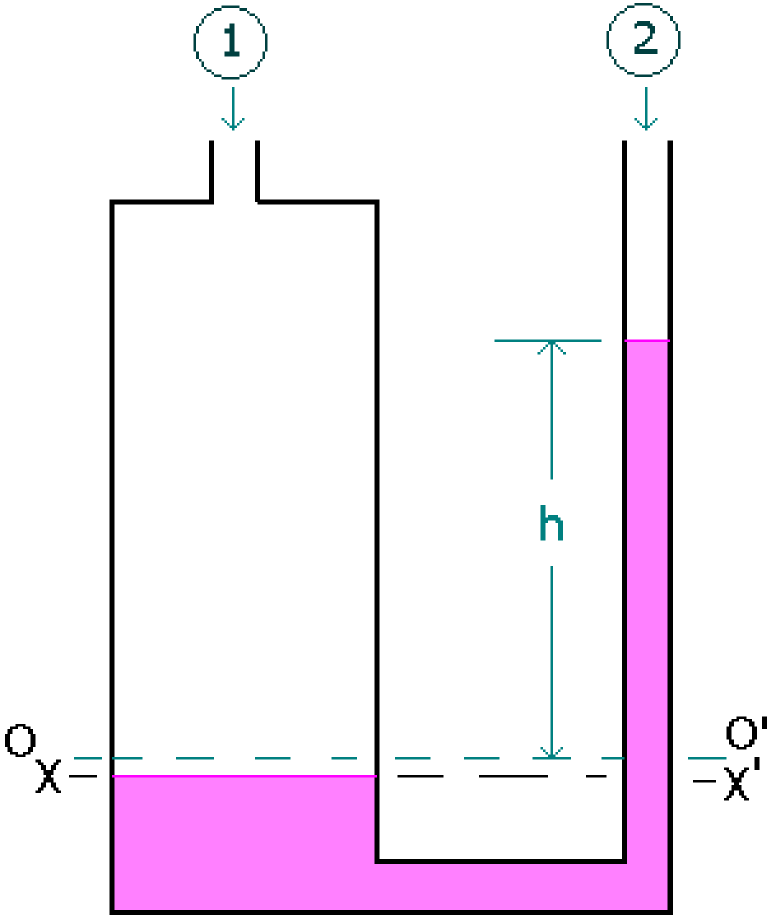
$$p_1 - p_2 = (\rho_m - \rho) g\, h$$

Inverted U - tube manometer



$$p_1 - p_2 = (\rho - \rho_m) g\, h$$

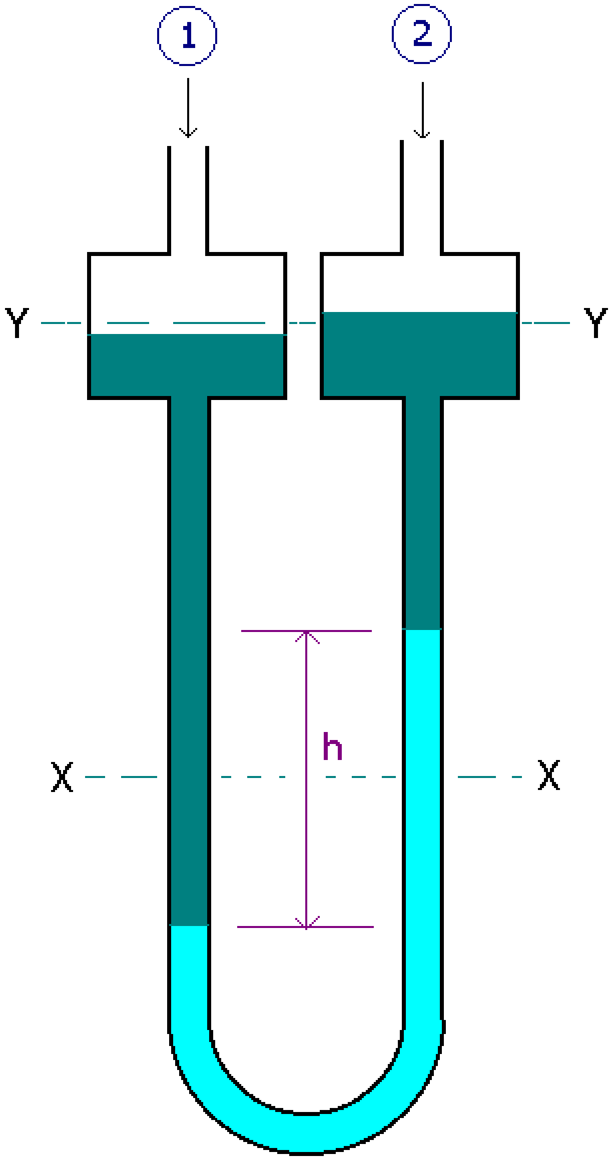
U - tube with one leg enlarged



used for more accurate readings

$$p_1 - p_2 = (\rho_m - \rho) g\, h$$

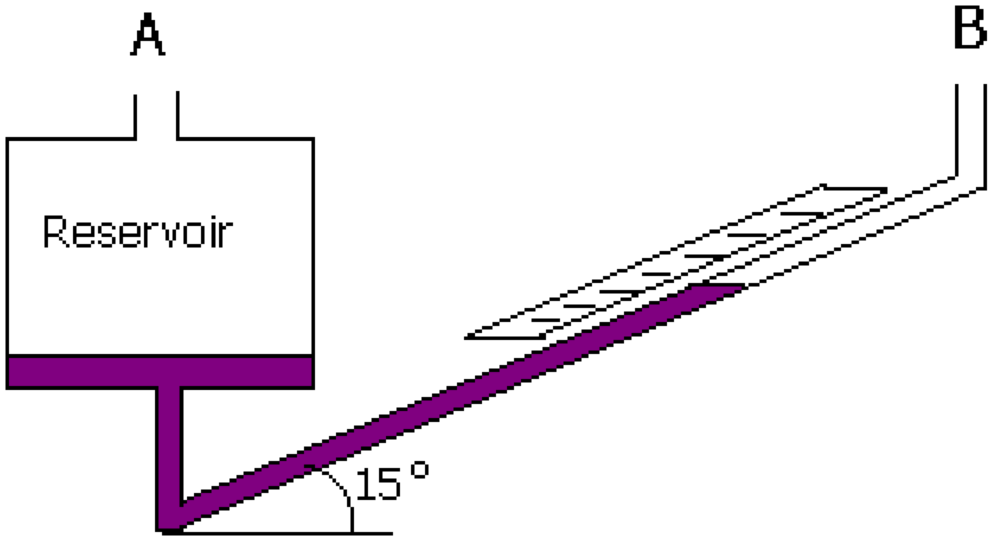
Two fluid U-tube manometer



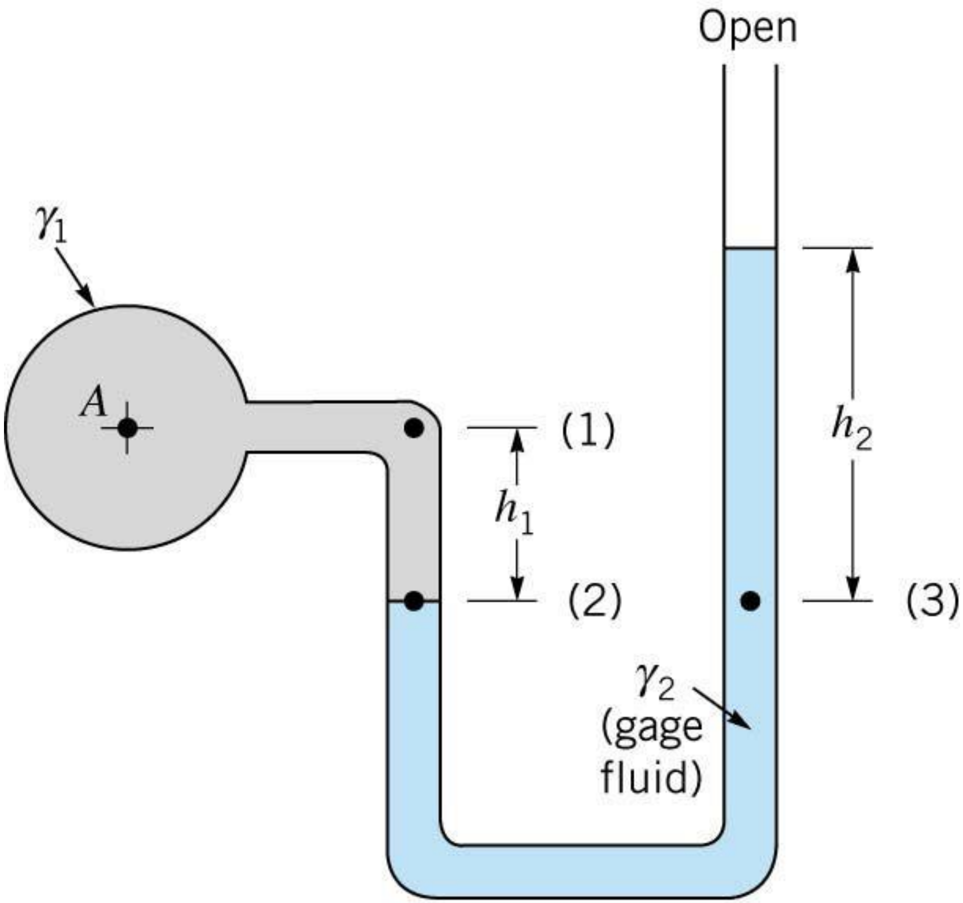
used for more accurate readings of small pressure differences

Inclined U-tube manometer

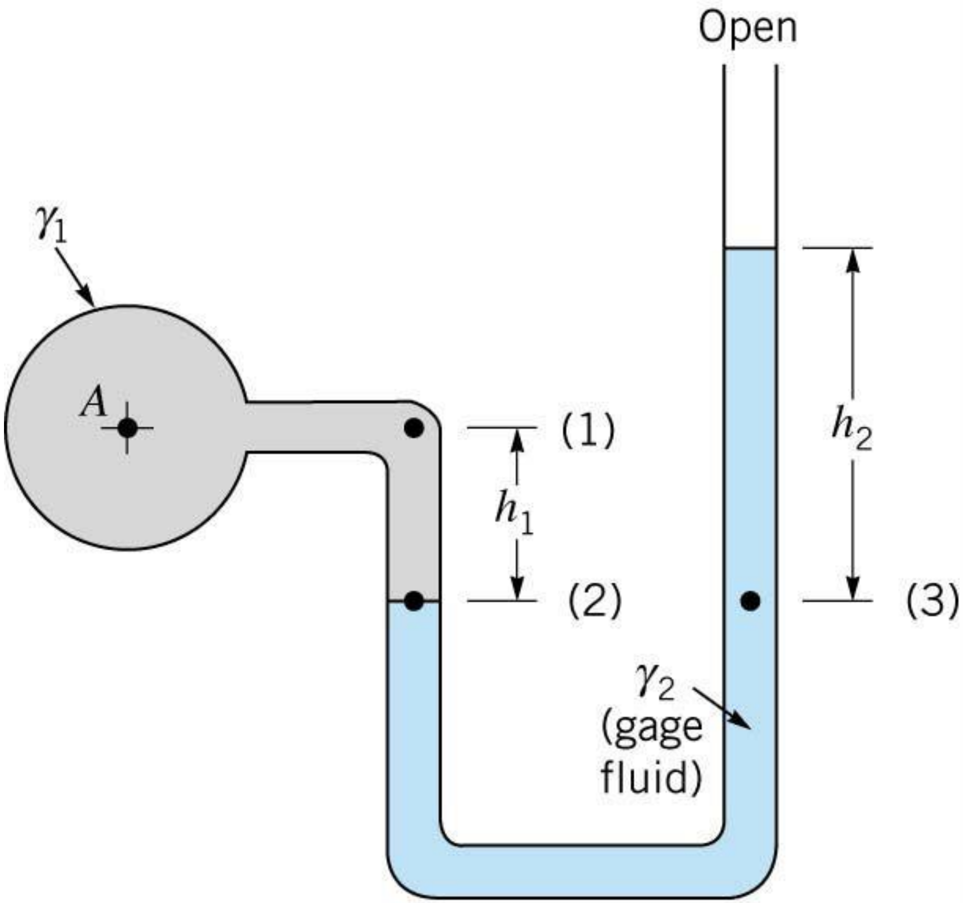
used for more accurate readings of small pressure differences



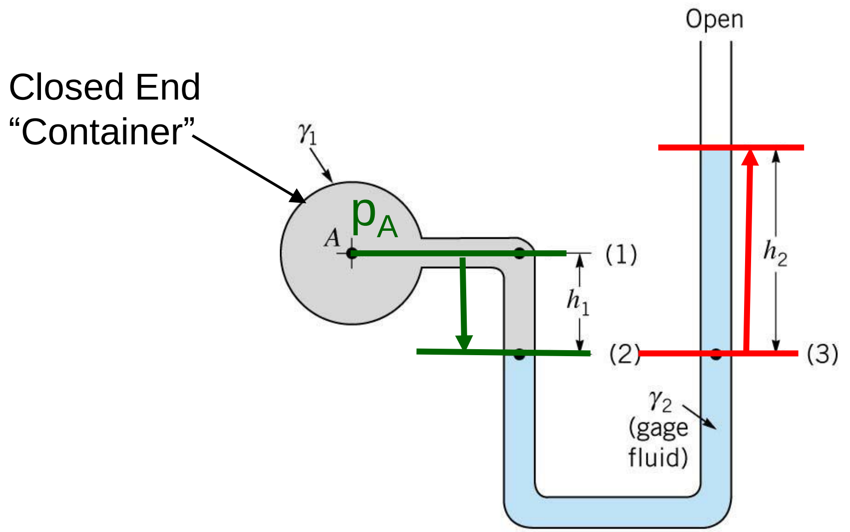
Measurement of Pressure: U-Tube Manometer



Measurement of Pressure: U-Tube Manometer



Measurement of Pressure: U-Tube Manometer



Note: in the same fluid we can “jump” across from 2 to 3 as they are at the sam level, and thus must have the same pressure.

The fluid in the U-tube is known as the gage fluid. The gage fluid type depends on the application, i.e. pressures attained, and whether the fluid measured is a gas or liquid.

Since, one end is open we can work entirely in gage pressure:

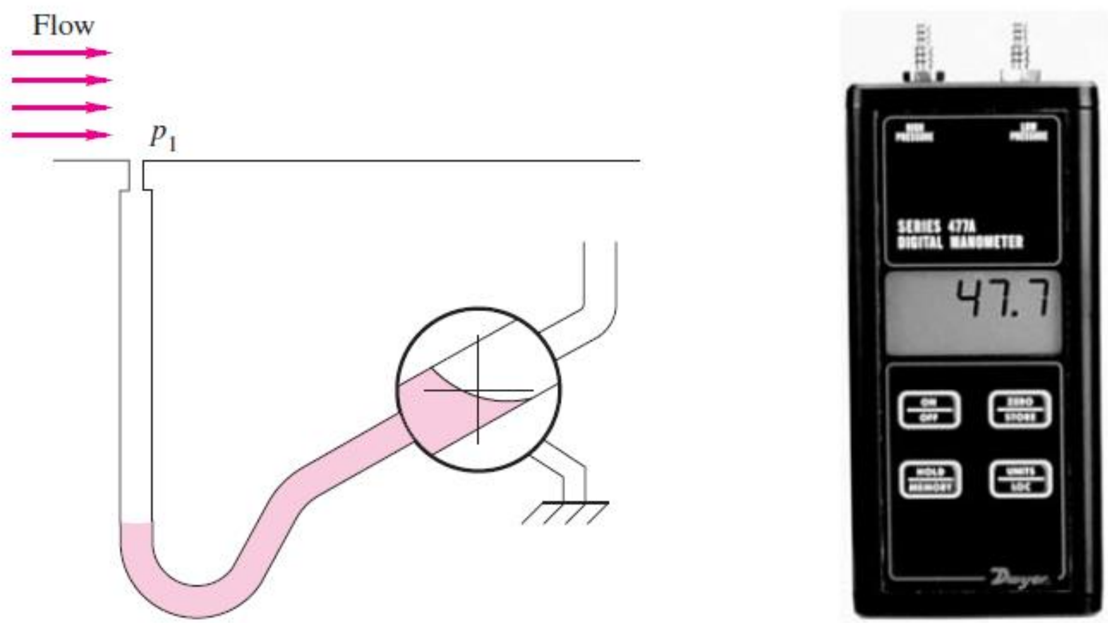
Moving from left to right: $p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$

Then the equation for the pressure in the container is the following:

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

If the fluid in the container is a gas, then the fluid 1 terms can be ignored:

$$p_A = \gamma_2 h_2$$



(*a*) (*b*)

Fig. 2.25 Two types of accurate manometers for precise measurements: (*a*) tilted tube with eyepiece; (*b*) a capacitive-type digital manometer of rated accuracy ±0.1 percent. *(Courtesy of Dwyer Instruments, Inc.)*

Pressure Instruments

Types of Pressure Instruments

- **Pressure Gauges (Vacuum, Compound, Absolute, Gauge)**
- **Differential Pressure Gauge**
- **Pressure Switch (Vacuum, Absolute, Gauge)**
- **Differential Pressure Switch**
- **Pressure Transmitter (Vacuum, Absolute, Gauge)**
- **Differential Pressure Transmitter**

PRESSURE GAUGE

PRESSURE SWITCH

DIFFERENTIAL PRESSURE TRANSMITTER

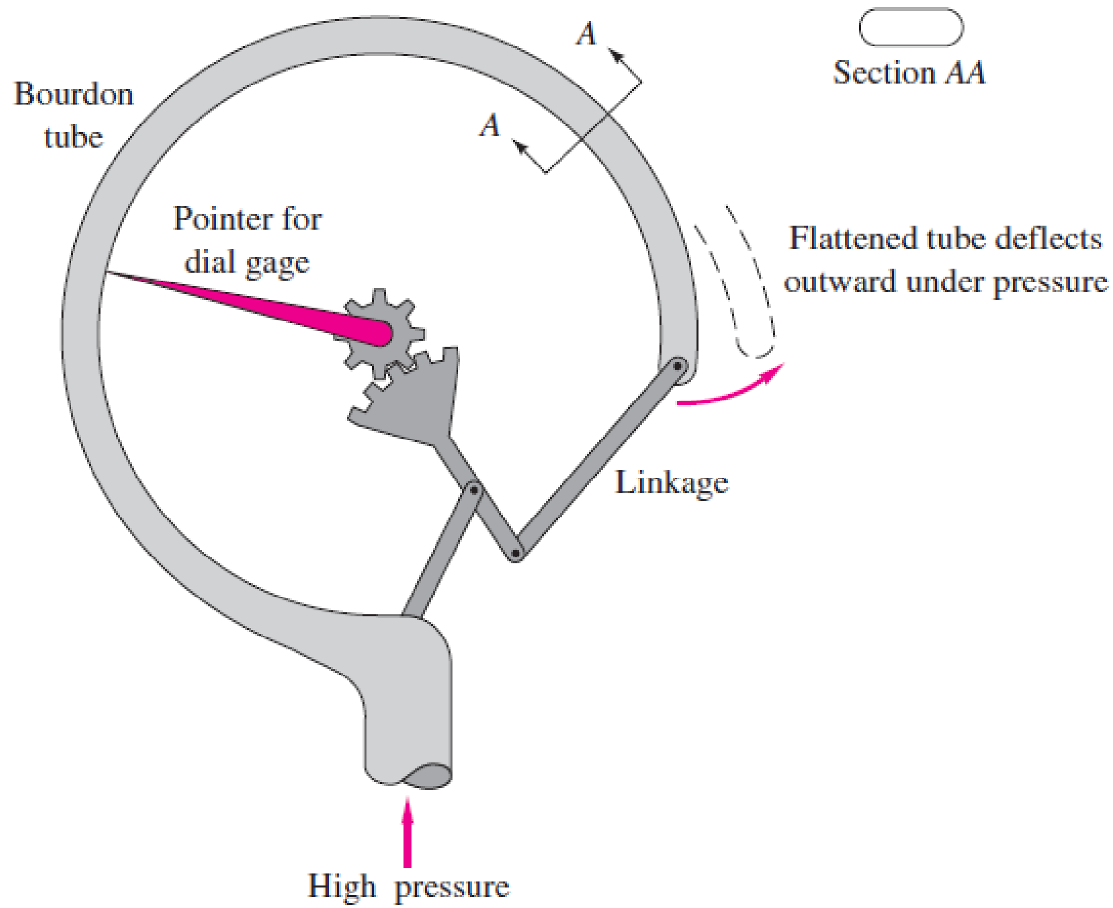
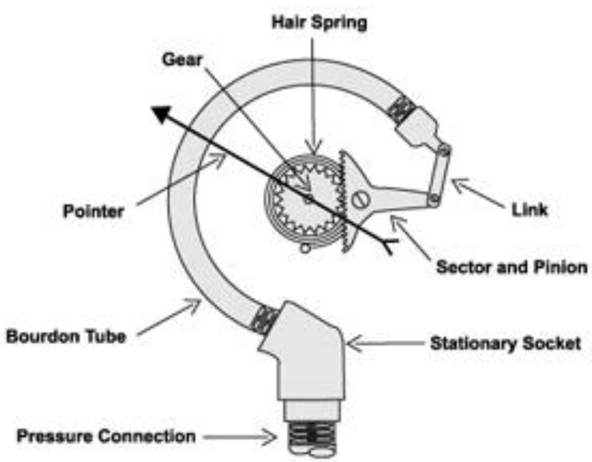


Fig. 2.26 Schematic of a bourdon-tube device for mechanical measurement of high pressures.

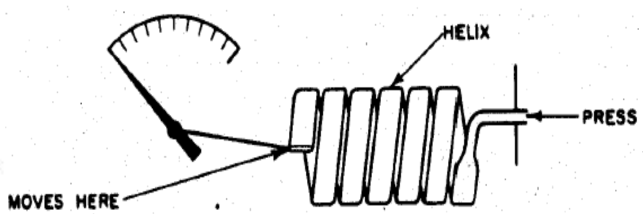
Pressure Gauge

Measuring Principle

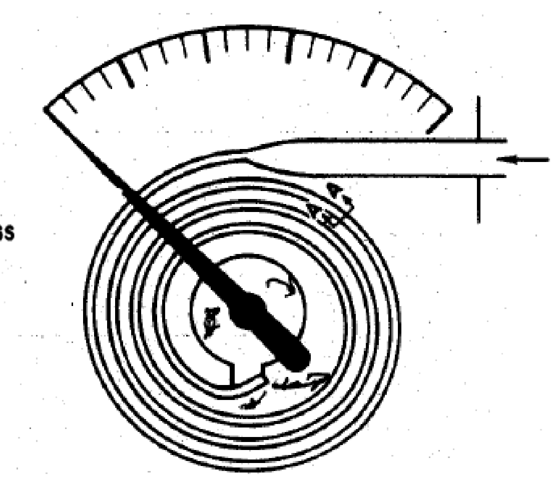
Bourdon tube measuring element is made of a thin-walled C-shape tube or spirally wound helical or coiled tube. When pressure is applied to the measuring system through the pressure port (socket), the pressure causes the Bourdon tube to straighten itself, thus causing the tip to move. The motion of the tip is transmitted via the link to the movement which converts the linear motion of the bourdon tube to a rotational motion that in turn causes the pointer to indicate the measured pressure.



"C" Type Bourdon



Helical Bourdon

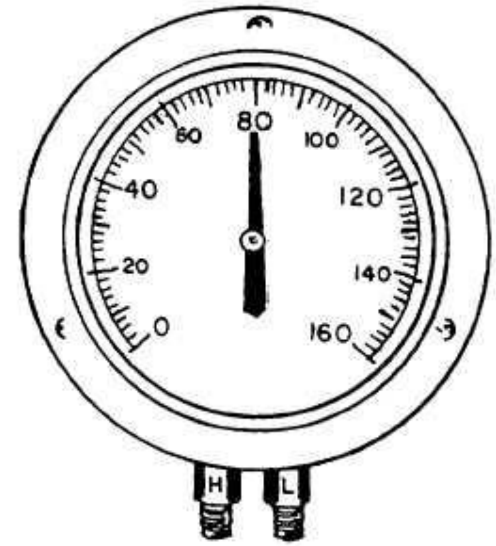


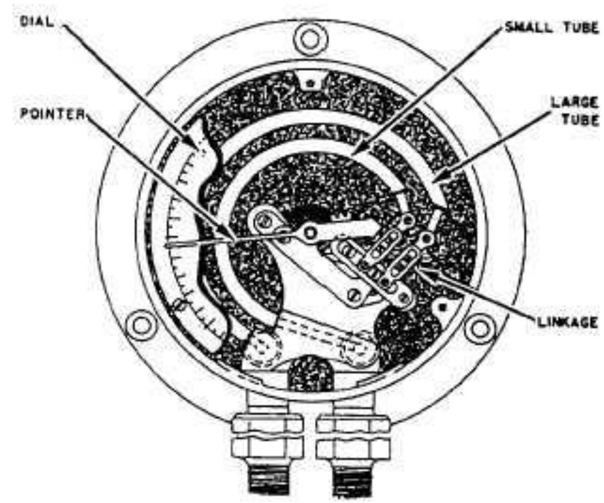
Coiled Bourdon

Differential Pressure Gauge

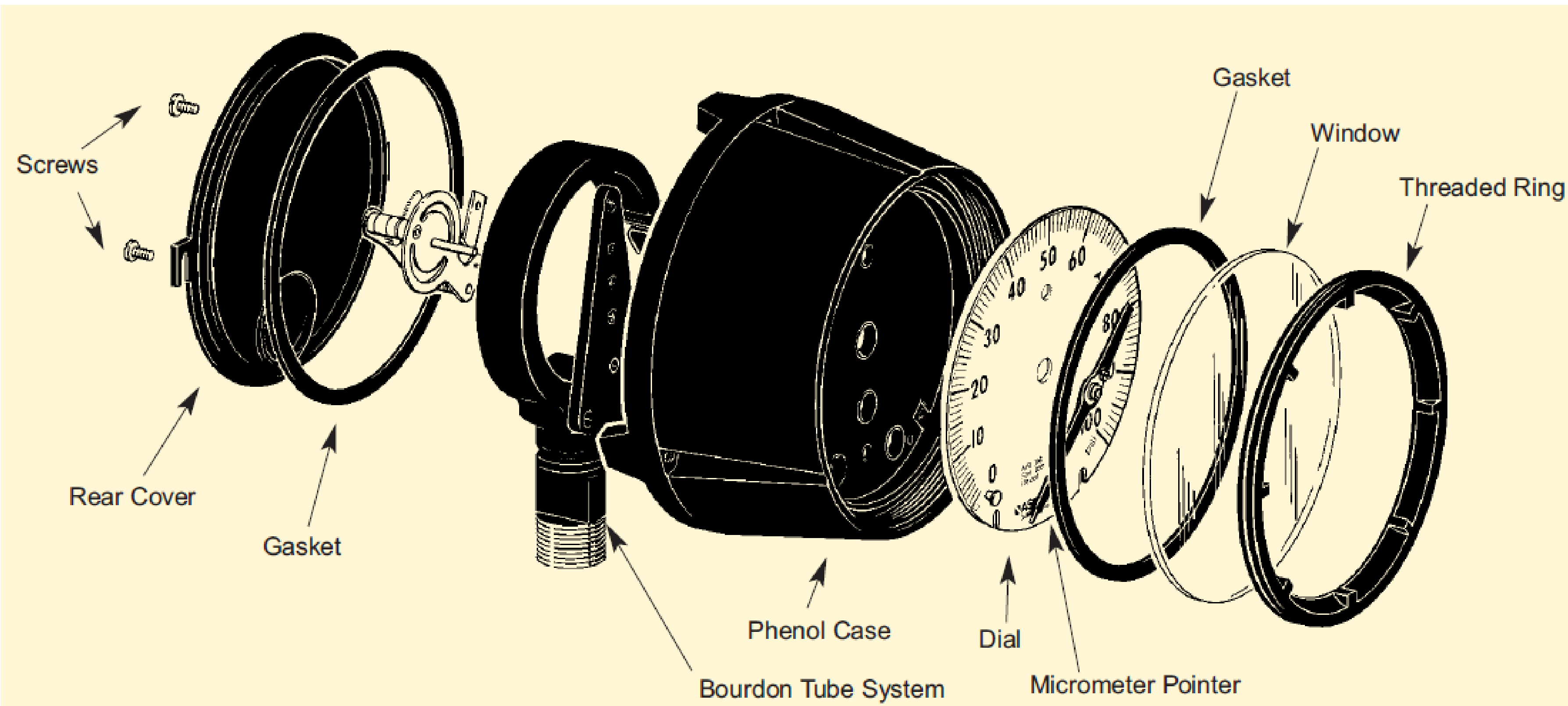
Measuring Principle:

- **Differential pressure gauges have two inlet ports, each connected to one of the volumes whose pressure is to be monitored.**
- **In cases where either input can be higher or lower than the other, a bi-directional differential range should be used.**





Pressure Gauge



Differential Pressure Gauge

Unidirectional and Bidirectional DP Gauges

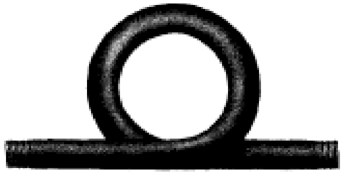
Pressure Gauge - Accessories

Pressure Limit Valve

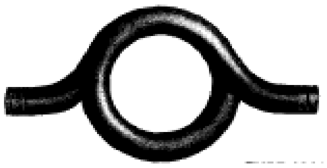
Protects pressure instruments against surges and pulsations. Provides automatic positive protection and accurate, repeatable performance. Automatic pressure shut-off, built in snubber enhances instrument protecting performance.

Siphon Tubes

Used to dissipate heat by trapping condensed liquid to keep high temperature steam or condensing vapor from damaging the pressure gauge.

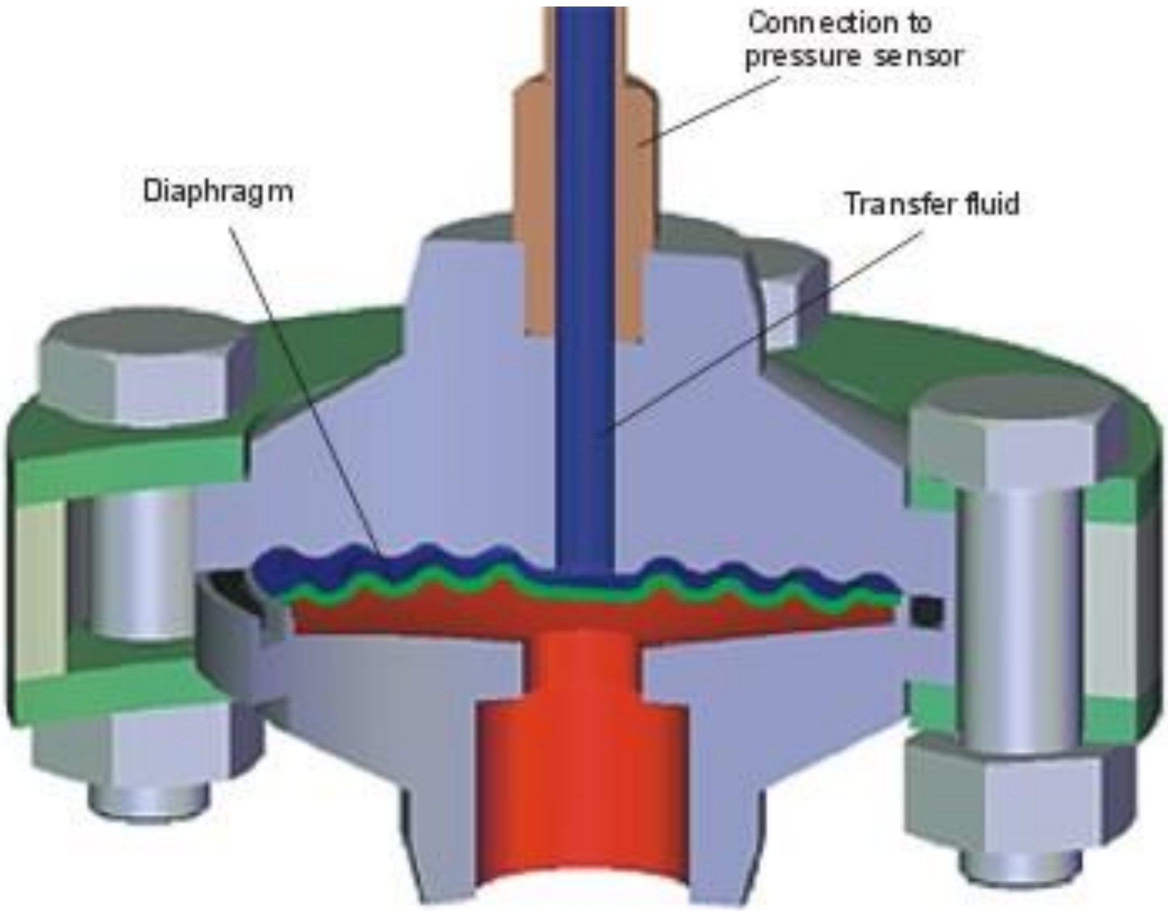


PIG TAIL



COIL PIPE

Diaphragm Seals



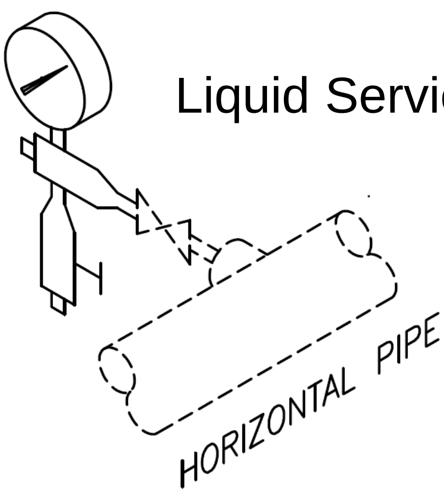
Pressure Gauge Selection Guideline

When selecting a Pressure Gauge, care should be given to a number of parameters which have an effect on the gauge's accuracy, safety, and cost.

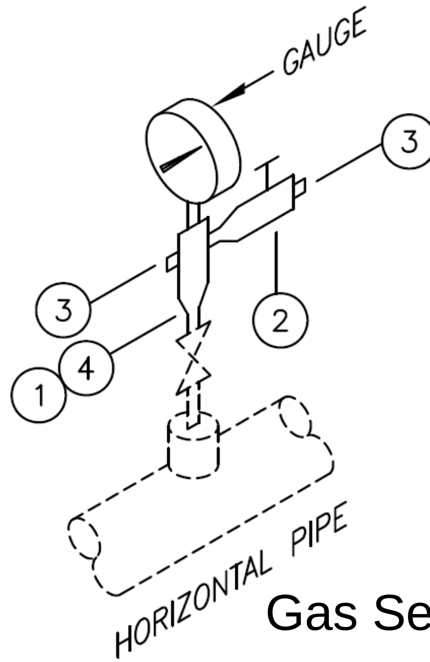
- Accuracy required
- Dial size
- Operating pressure range
- Chemical compatibility with gauge construction materials
- Operating temperature range
- Vibration, pulsation, and shock
- Pressure fluid composition
- Mounting requirement

Pressure Gauge Installation

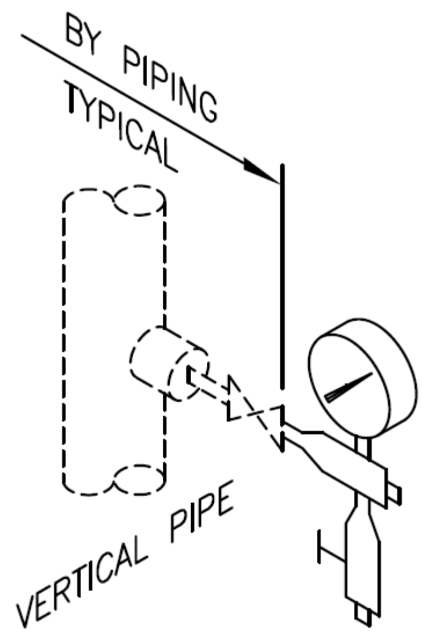
- **Top connection preferred for gas installations & side connection preferred for liquid installations.**
- **The pressure gauge can be connected to the pipe by individual block and bleed valves or a two way manifold.**



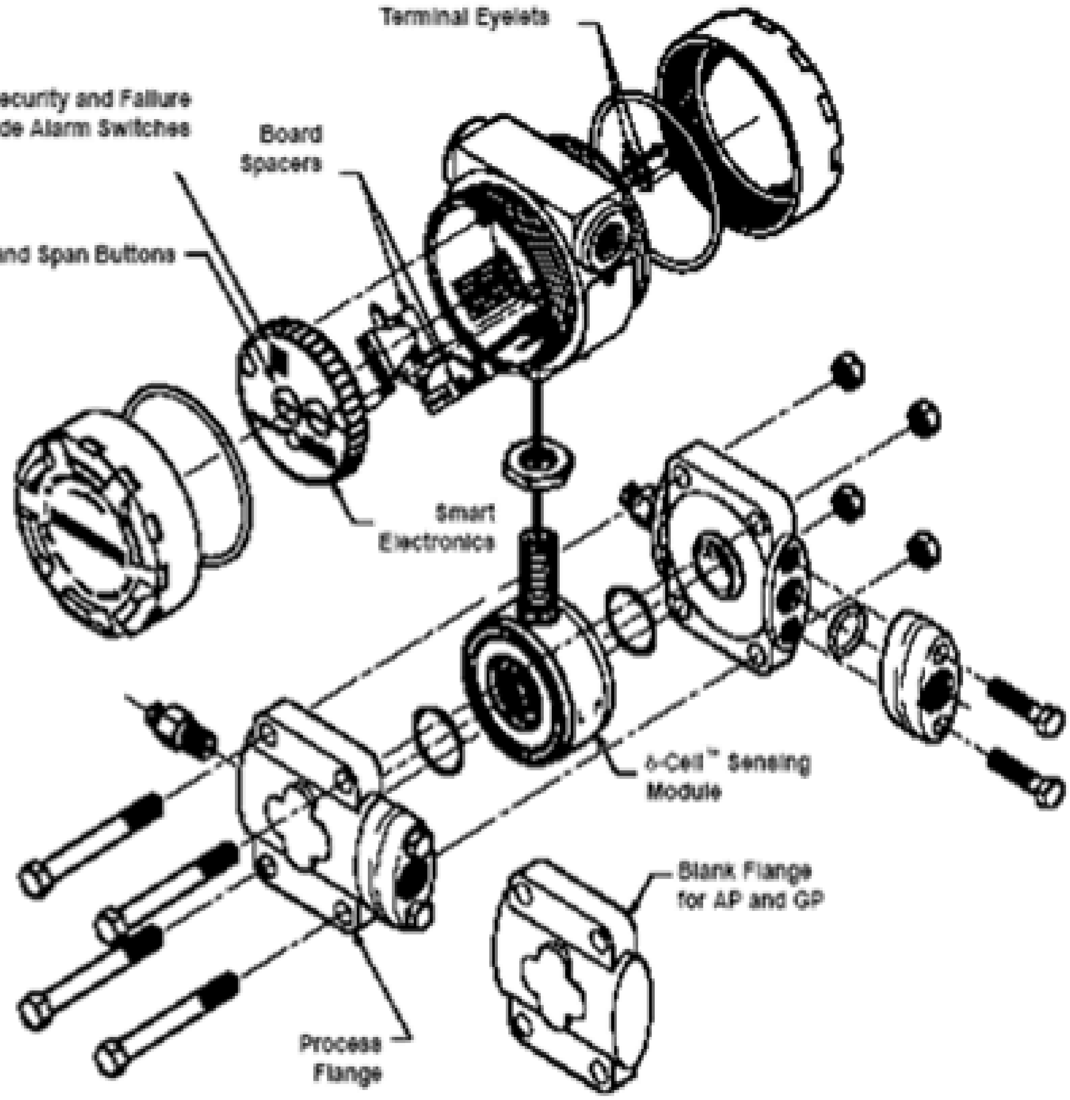
Liquid Service



Gas Service



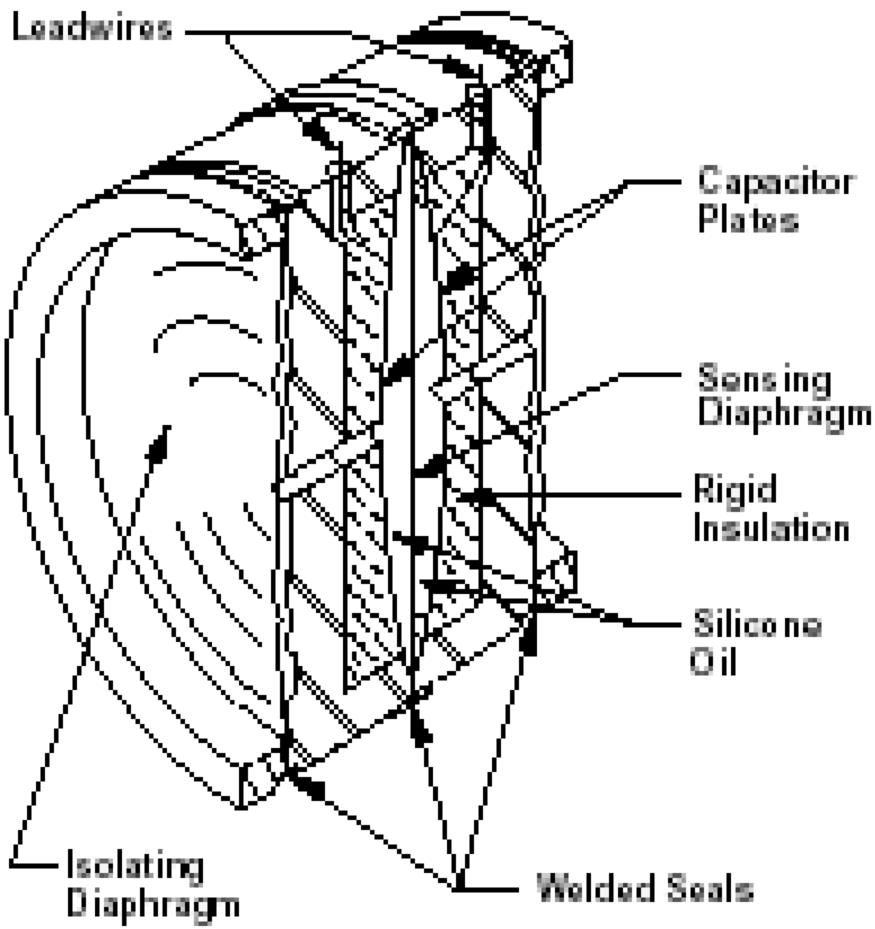
Pressure Transmitter



Pressure Transmitter

- A Pressure Transmitter is used where indication and/or record of pressure is required at a location not adjacent to the primary element.
- A Pressure Transmitter is used for both indication and control of a process.
- A Pressure Transmitter is used where overall high performance is mandatory.
- Both Electronic and Pneumatic Transmitters are used.
- These can be either Gauge, Absolute or Differential Pressure Transmitters.

Transmitter Measuring Principle



- The diagram shows an electronic differential pressure sensor. This particular type utilizes a two-wire capacitance technique.
- Another common measuring technique is a strain gauge.
- Process pressure is transmitted through isolating diaphragms and silicone oil fill fluid to a sensing diaphragm.
- The sensing diaphragm is a stretched spring element that deflects in response to the differential pressure across it.
- The displacement of the sensing diaphragm is proportional to the differential pressure.
- The position of the sensing diaphragm is detected by capacitor plates on both sides of the sensing diaphragm.
- The differential capacitance between the sensing diaphragm and the capacitor plates is converted electronically to a 4–20 mA or 1-5 VDC signal.
- For a gauge pressure transmitter, the low pressure side is referenced to atmospheric pressure.

Pressure Transmitter

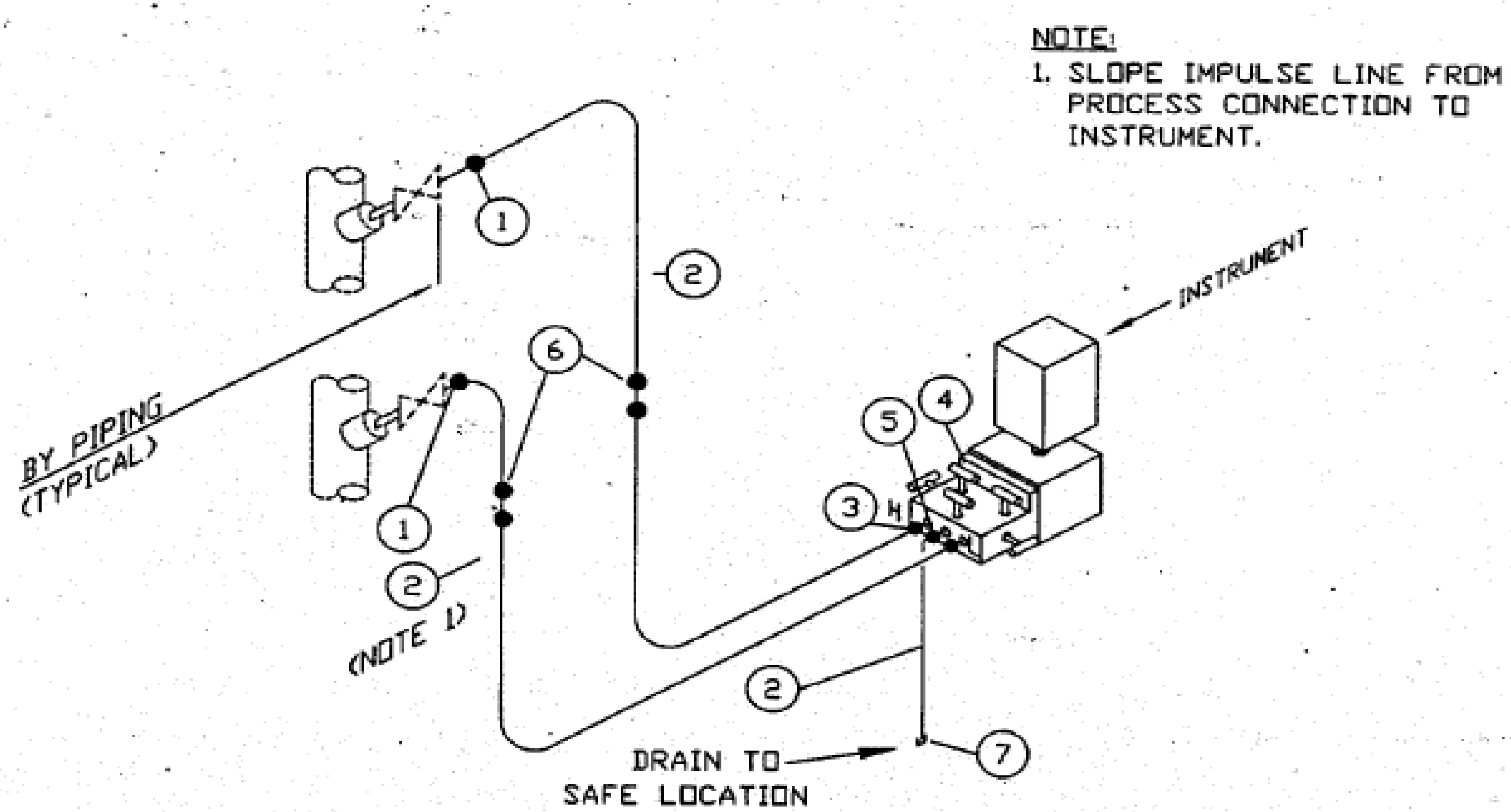
- Typical Outputs
  - 4 to 20 milliamp (mA). analog signal
  - Smart HART digital signal (superimposed on analog signal)
  - Fieldbus digital signal
  - 3 to15 psi pneumatic signal

Diaphragm Seal System

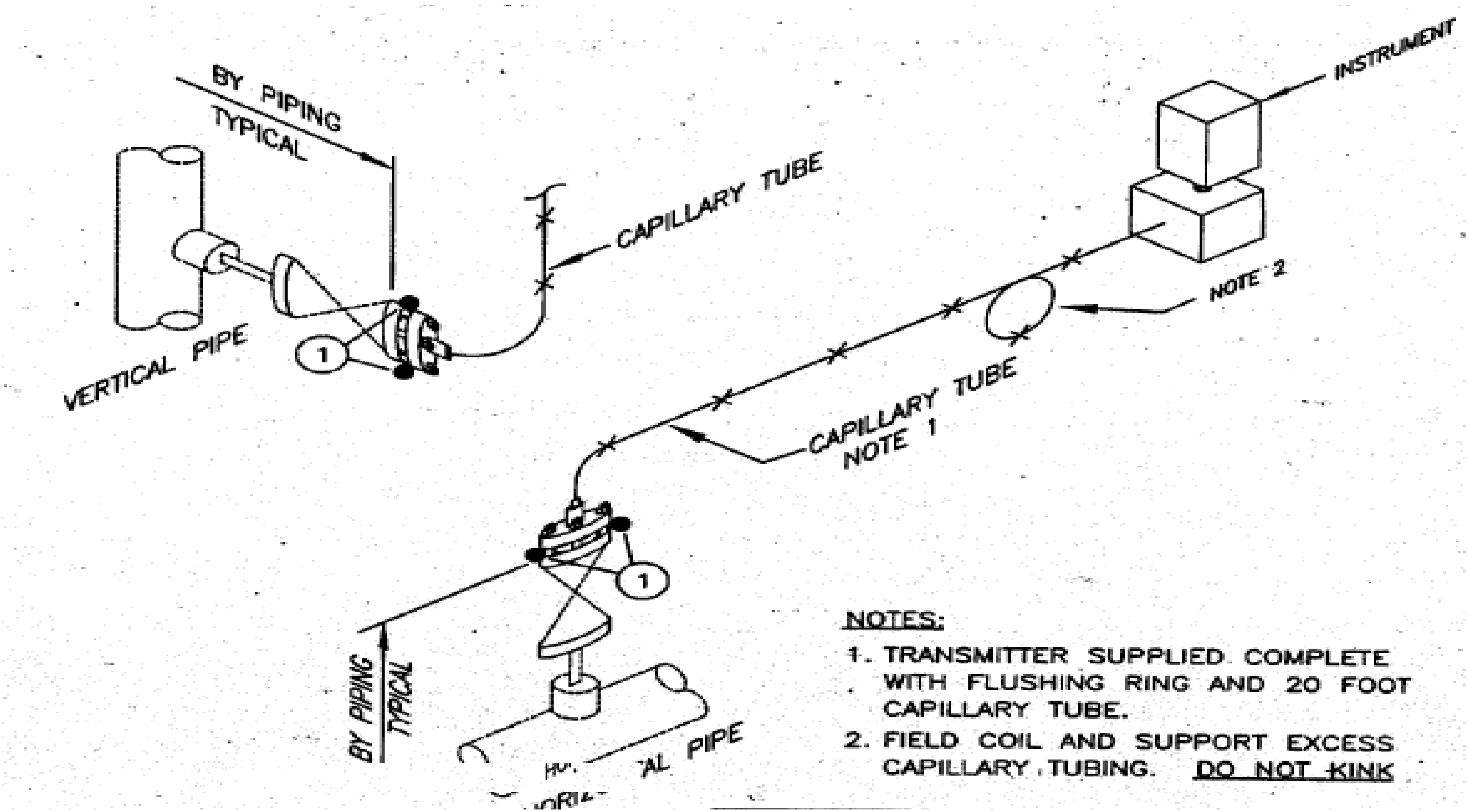
- A diaphragm seal system consists of a pressure transmitter, diaphragm seals, a fill fluid, and either a direct mount or capillary style connection.
- During operation, the thin, flexible diaphragm and fill fluid separate the pressure sensitive element of the transmitter from the process medium. The capillary tubing or direct mount flange connects the diaphragm to the transmitter.
- When process pressure is applied, the diaphragm transfers the measured pressure through the filled system and capillary tubing to the transmitter element.
- This transferred pressure displaces the sensing diaphragm in the pressure-sensitive element of the transmitter.
- The displacement is proportional to the process pressure and is electronically converted to an appropriate current, voltage, or digital HART output signal.

Pressure Transmitter installation

- Mounting above tap is typical for gas service and mounting below tap typical for liquid and steam services.
- Direct mount is possible for low temperature services.



Diaphragm Seal installation



Pressure Instruments

Selection of Pressure Instruments: Rules of Thumb:

- Application: Understand your application. Examine the particulars of your application. Is it necessary to know if the pressure is negative or positive? Do you need to know the difference in pressure between two points? Answering these questions about your application will go a long way in helping select the right pressure transmitter.
- Wetted Parts: Selecting the transmitter with wetted parts that are compatible with the medium to be measured helps to ensure a long-lasting measurement solution.