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توزیع فشار در سیال ساکن

2.3 Hydrostatic Pressure Distributions

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$$\nabla p = \rho\mathbf{g} \tag{2.15}$$

This is a *hydrostatic* distribution and is correct for all fluids at rest, regardless of their viscosity, because the viscous term vanishes identically.

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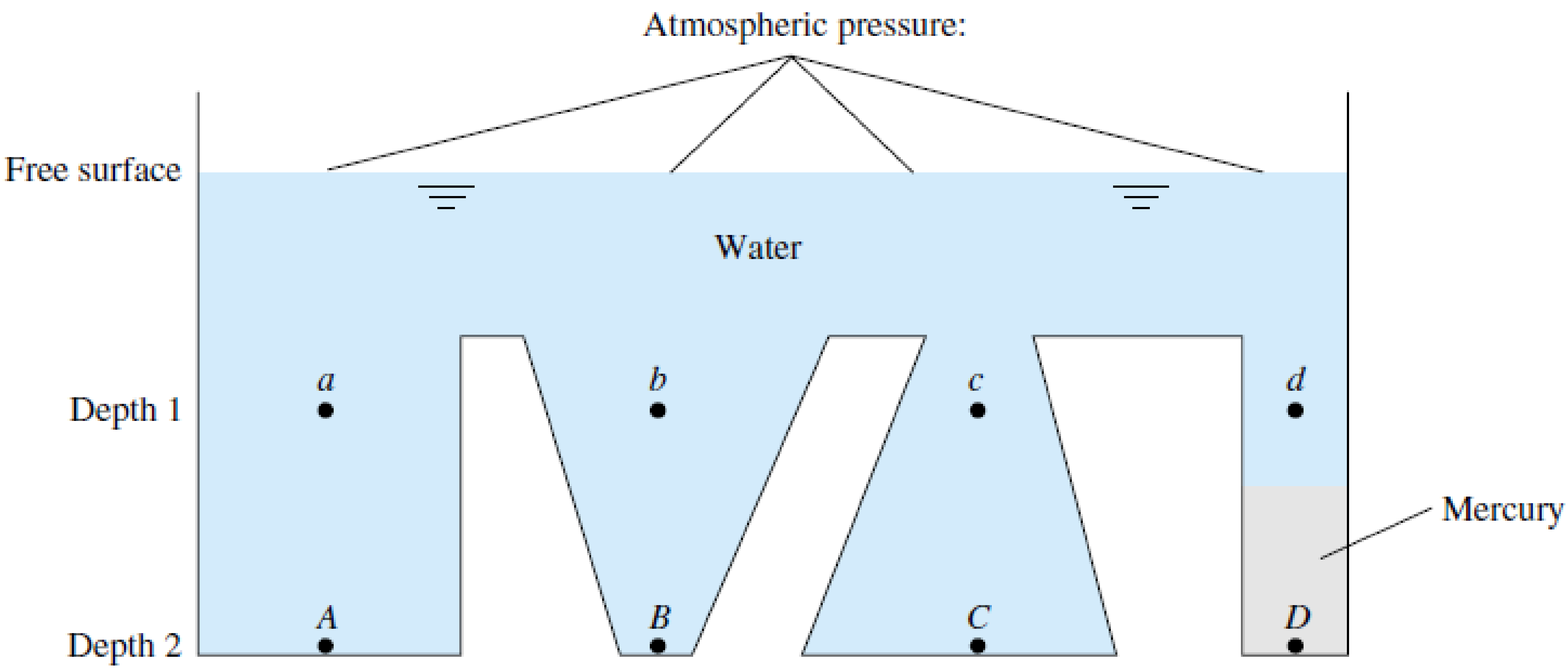
In our customary coordinate system z is "up." Thus the local-gravity vector for small-scale problems is

$$\mathbf{g} = -g\mathbf{k} \tag{2.16}$$

where g is the magnitude of local gravity, for example, 9.807 m/s^2. For these coordinates Eq. (2.15) has the components

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\rho g = -\gamma \tag{2.17}$$



Atmospheric pressure:
Free surface
Water
a
b
c
d
Depth 1
Mercury
A
B
C
D
Depth 2

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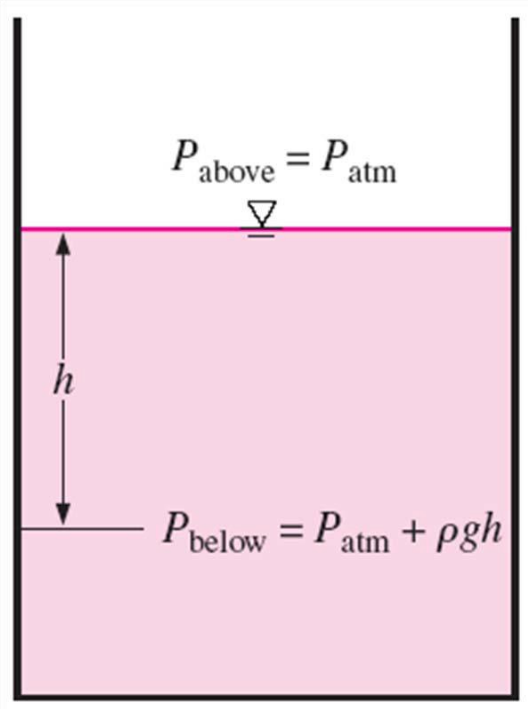
$$p_2 - p_1 = -\int_1^2 \gamma \, dz \tag{2.18}$$



Pressure in a liquid at rest increases linearly with distance from the free surface

Effect of Variable Gravity

For a spherical planet of uniform density, the acceleration of gravity varies inversely as the square of the radius from its center

$$g = g_0\left(\frac{r_0}{r}\right)^2 \tag{2.19}$$

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where r_0 is the planet radius and g_0 is the surface value of g. For earth, $r_0 \approx 3960$ statute mi ≈ 6400 km. In typical engineering problems the deviation from r_0 extends from the deepest ocean, about 11 km, to the atmospheric height of supersonic transport operation, about 20 km. This gives a maximum variation in g of $(6400/6420)^2$, or 0.6 percent. We therefore neglect the variation of g in most problems.

Hydrostatic Pressure in Liquids

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$$z_1 - z_2 = \frac{p_2}{\gamma} - \frac{p_1}{\gamma}$$

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γ is called the *specific weight*

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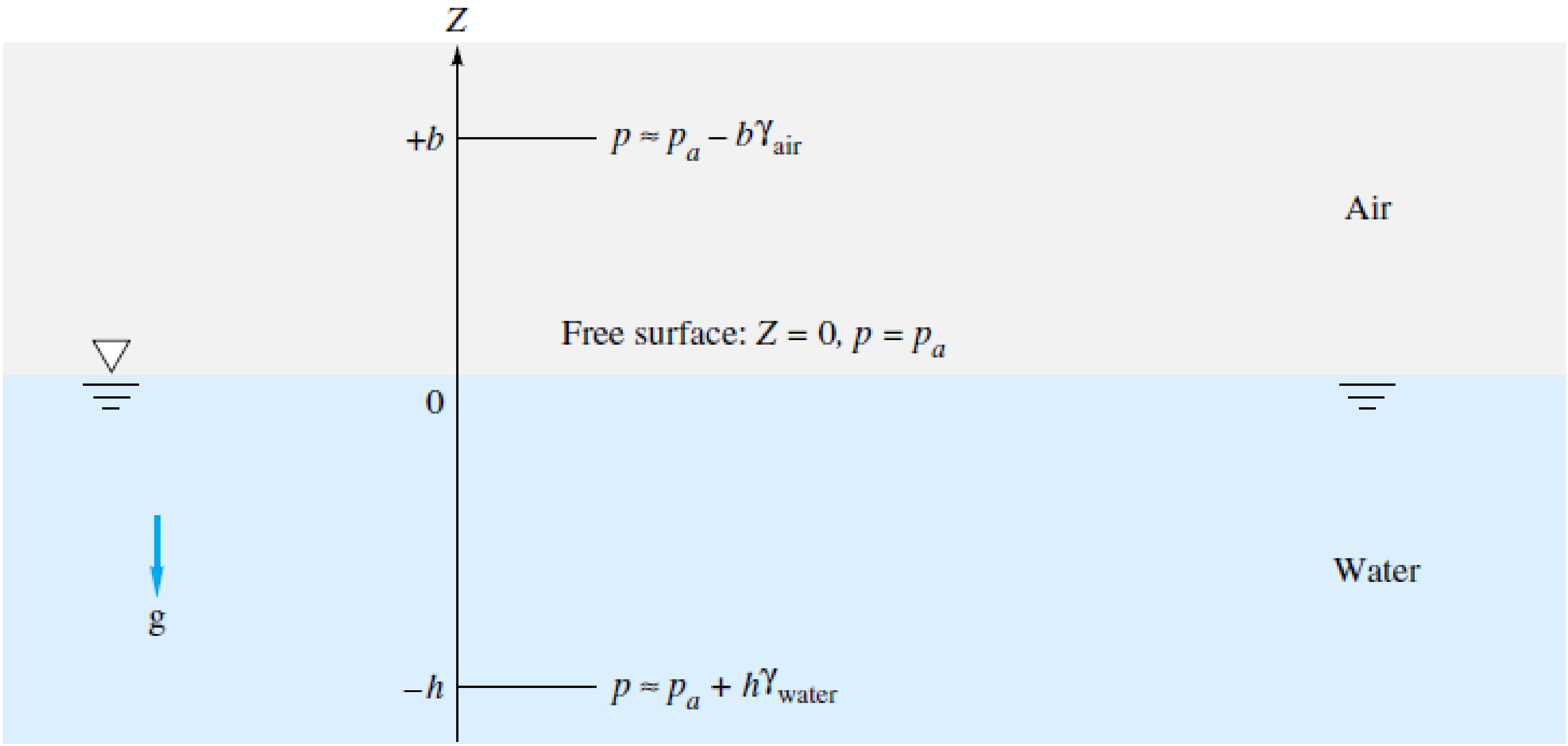
γ is called the *specific weight*

p/γ is a length called the *pressure head*

Fluid	Specific weight γ at 68°F = 20°C	
	lbf/ft^3	N/m^3
Air (at 1 atm)	0.0752	11.8
Ethyl alcohol	49.2	7,733
SAE 30 oil	55.5	8,720
Water	62.4	9,790
Seawater	64.0	10,050
Glycerin	78.7	12,360
Carbon tetrachloride	99.1	15,570
Mercury	846	133,100


Z
$+b$
$p \approx p_a - b\gamma_{\text{air}}$
Air
Free surface: $Z = 0$, $p = p_a$
0
Water
g
$-h$
$p \approx p_a + h\gamma_{\text{water}}$

EXAMPLE 2.1

Newfound Lake, a freshwater lake near Bristol, New Hampshire, has a maximum depth of 60 m, and the mean atmospheric pressure is 91 kPa. Estimate the absolute pressure in kPa at this maximum depth.

Solution

From Table 2.1, take $\gamma \approx 9790\ \mathrm{N/m^3}$. With $p_a = 91$ kPa and $z = -60$ m, Eq. (2.21) predicts that the pressure at this depth will be

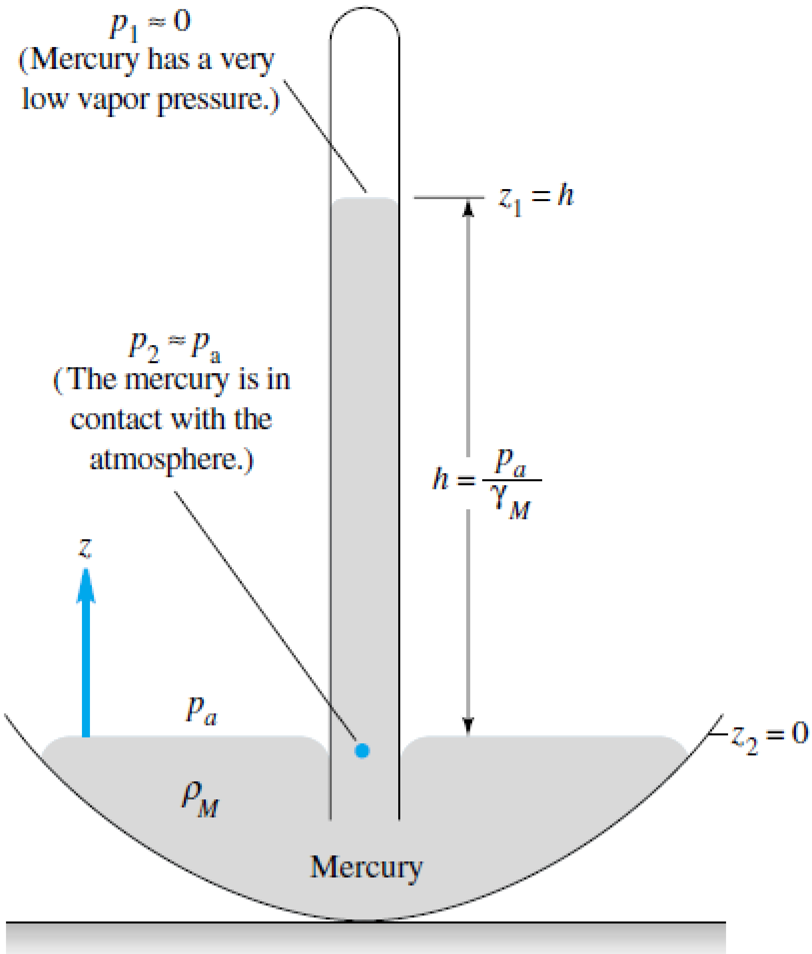
$$p = 91\ \mathrm{kN/m^2} - (9790\ \mathrm{N/m^3})(-60\ \mathrm{m})\,\frac{1\ \mathrm{kN}}{1000\ \mathrm{N}}$$

$$= 91\ \mathrm{kPa} + 587\ \mathrm{kN/m^2} = 678\ \mathrm{kPa} \qquad \textit{Ans.}$$

By omitting p_a we could state the result as $p = 587$ kPa (gage).

The Mercury Barometer

The Mercury Barometer



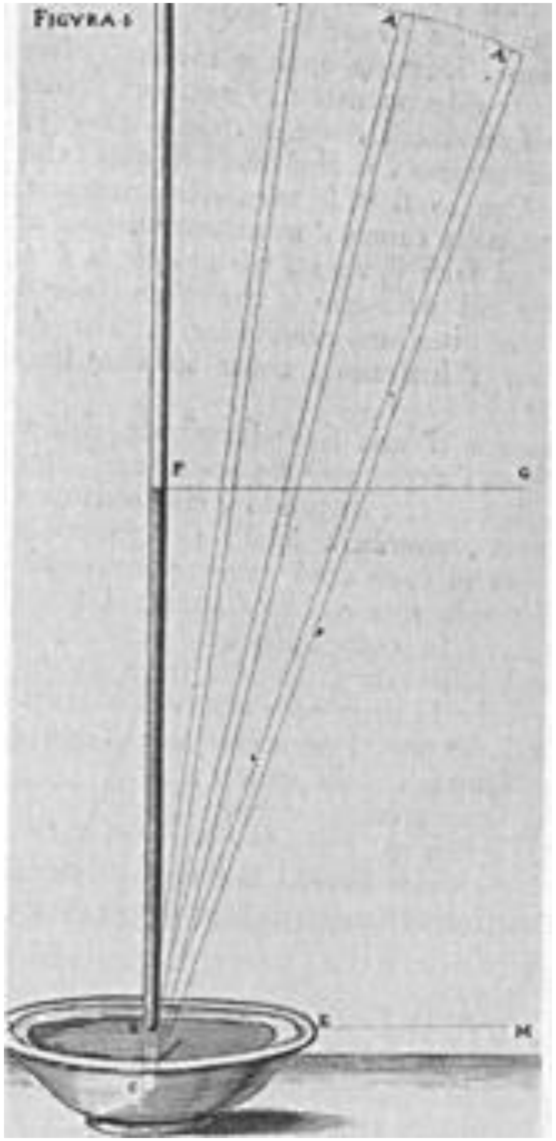
(a)

Evangelista Torricelli
(1608-1647)

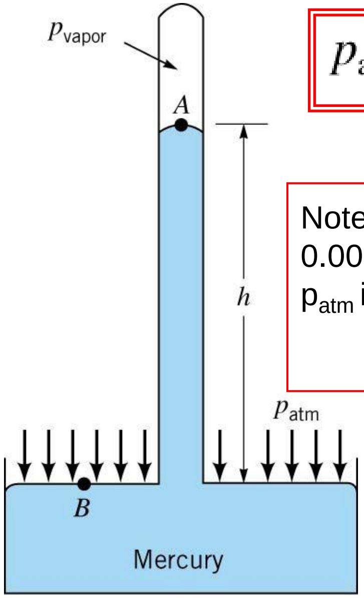
Measurement of Pressure: Barometers

The first mercury barometer was constructed in 1643-1644 by Torricelli. He showed that the height of mercury in a column was 1/14 that of a water barometer, due to the fact that mercury is 14 times more dense that water. He also noticed that level of mercury varied from day to day due to weather changes, and that at the top of the column there is a vacuum.

Torricelli's Sketch



Schematic:

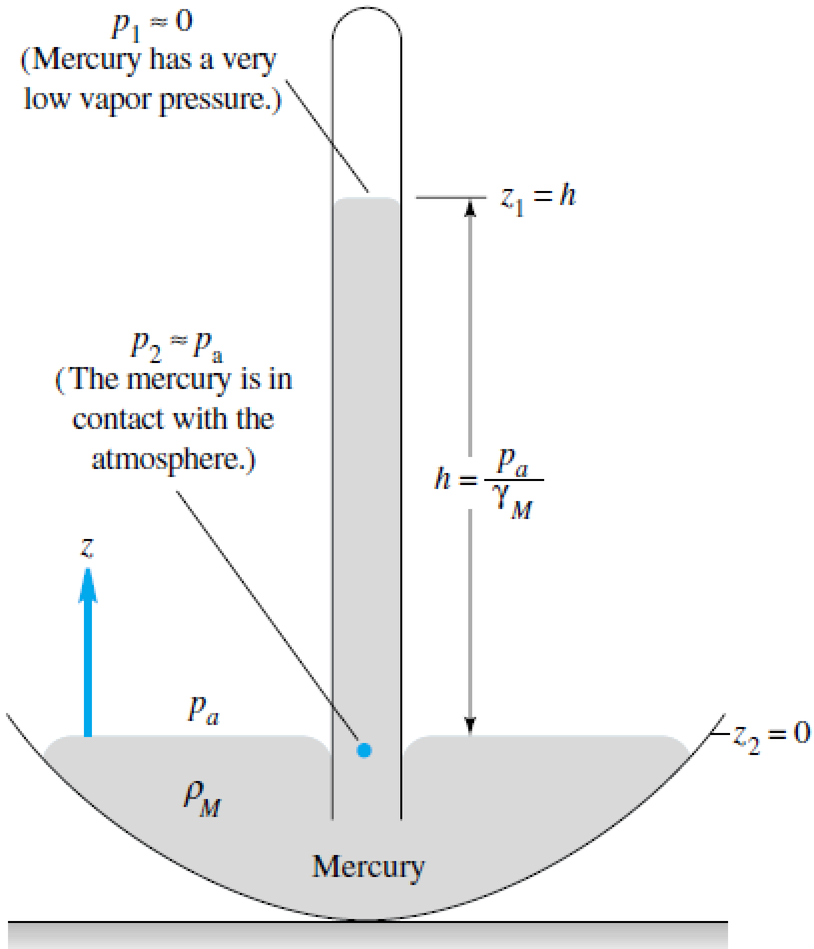


$$\boxed{p_{\text{atm}} = \gamma h + p_{\text{vapor}}}$$

Note, often p_{vapor} is very small, 0.0000231 psia at 68° F, and p_{atm} is 14.7 psi, thus:

$$p_{\text{atm}} \approx \gamma h$$

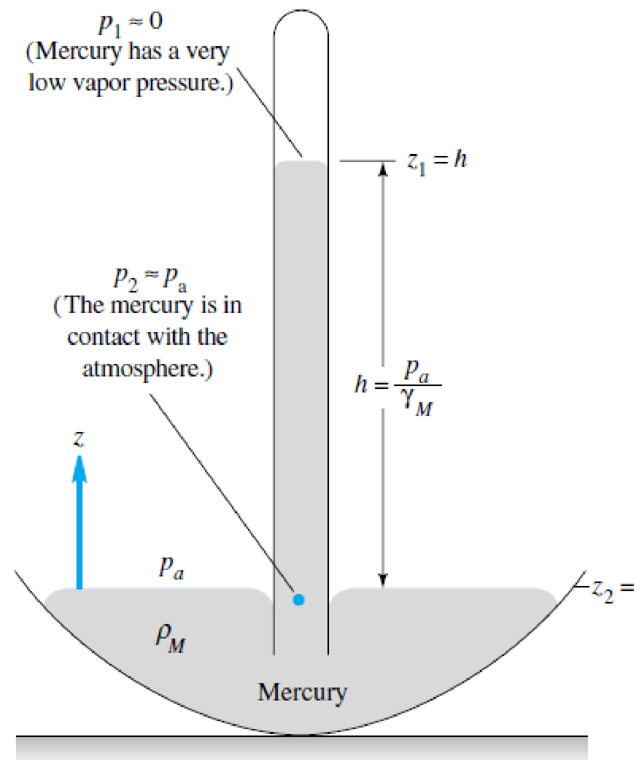
The Mercury Barometer



(a)

a modern portable barometer,

The Mercury Barometer



(a)

At sea-level standard, with $p_a = 101{,}350$ Pa and $\gamma_M = 133{,}100\ \mathrm{N/m^3}$ from Table 2.1, the barometric height is $h = 101{,}350/133{,}100 = 0.761$ m or 761 mm. In the United States the weather service reports this as an atmospheric "pressure" of 29.96 inHg (inches of mercury). Mercury is used because it is the heaviest common liquid. A water barometer would be 34 ft high.

Hydrostatic Pressure in Gases

$$p_2 - p_1 = -\int_1^2 \gamma \, dz \qquad (2.18)$$

Hydrostatic Pressure in Gases

Gases are compressible, with density nearly proportional to pressure. Thus density must be considered as a variable in Eq. (2.18) if the integration carries over large pressure changes. It is sufficiently accurate to introduce the perfect-gas law $p = \rho RT$ in Eq.

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT} g$$

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$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT} g$$

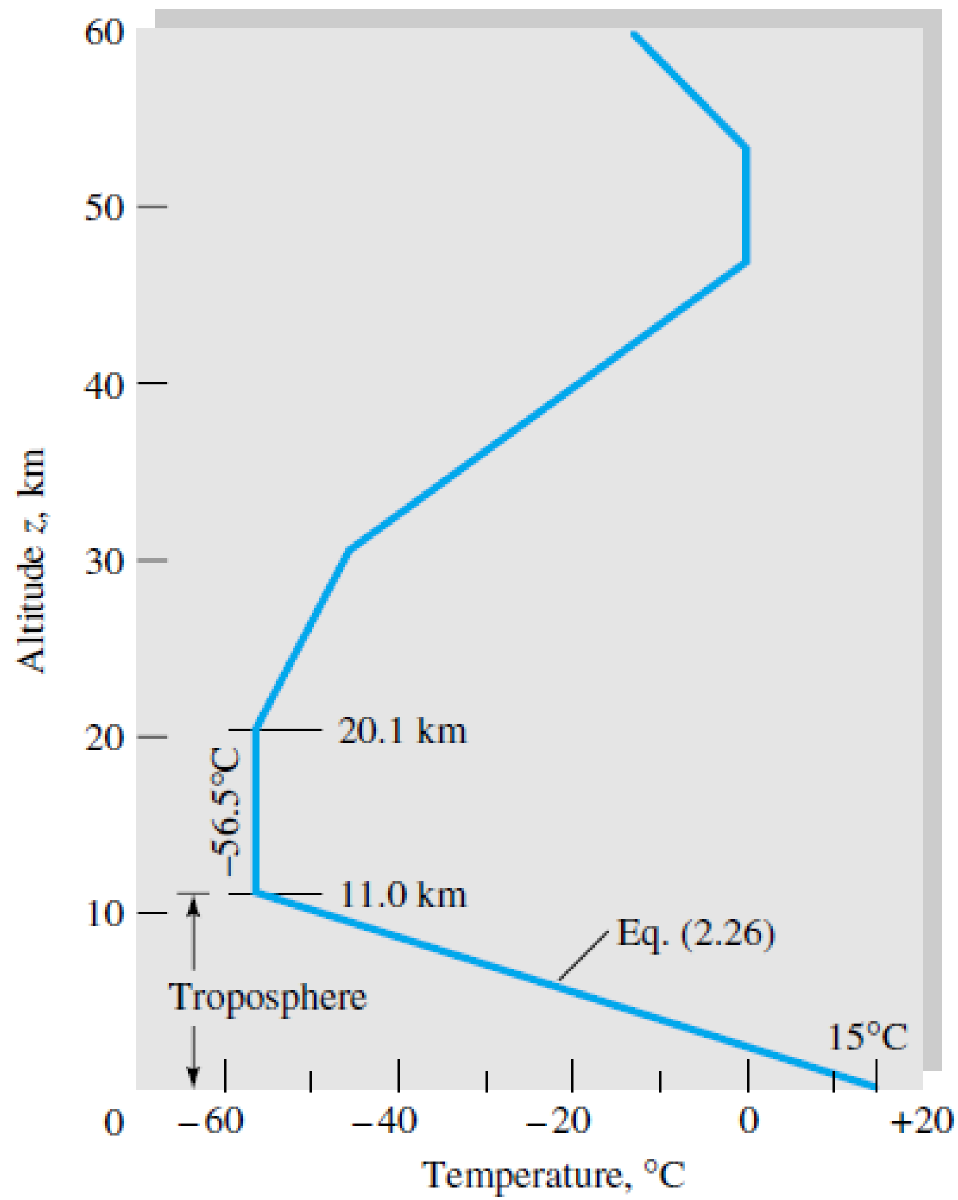
Separate the variables and integrate between points 1 and 2:

$$\int_1^2 \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_1^2 \frac{dz}{T}$$

$$\int_1^2 \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_1^2 \frac{dz}{T}$$

The integral over z requires an assumption about the temperature variation $T(z)$. One common approximation is the *isothermal atmosphere*, where $T = T_0$:

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0} \right] \tag{2.24}$$



60
50
40
30
20
10
0
Altitude z, km
20.1 km
−56.5°C
11.0 km
Eq. (2.26)
Troposphere
15°C
−60
−40
−20
0
+20
Temperature, °C

atmospheric temperature drops off nearly linearly with z up to an altitude of about 36,000 ft (11,000 m):

$$T \approx T_0 - Bz \tag{2.25}$$

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Here T_0 is sea-level temperature (absolute) and B is the *lapse rate*, both of which vary somewhat from day to day. By international agreement [1] the following standard values are assumed to apply from 0 to 36,000 ft:

$$T_0 = 518.69°\text{R} = 288.16\ \text{K} = 15°\text{C}$$

$$B = 0.003566°\text{R/ft} = 0.00650\ \text{K/m} \tag{2.26}$$

This lower portion of the atmosphere is called the *troposphere*. Introducing Eq. (2.25) into (2.23) and integrating, we obtain the more accurate relation

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$$p = p_a\left(1 - \frac{Bz}{T_0}\right)^{g/(RB)} \qquad \text{where } \frac{g}{RB} = 5.26 \text{ (air)} \tag{2.27}$$

Compressible fluid

- Gases are compressible i.e. their density varies with temperature and pressure $\rho = PM/RT$
  - For small elevation changes (as in engineering applications, tanks, pipes etc) we can neglect the effect of elevation on pressure
  - In the general case start from:

$$\frac{dP}{dz} = -\rho g$$

$$for \ \mathrm{T} = T_o = \mathrm{const}:$$

$$P_2 = P_1 \exp\left[-\frac{g\ M(z_2 - z_1)}{RT_o}\right]$$

Compressible

Linear Temperature Gradient

$$T = T_0 - \alpha(z - z_0)$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{gM}{R} \int_{z_0}^{z} \frac{dz}{T_0 - \alpha(z - z_0)}$$

$$p(z) = p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0} \right]^{gM/\alpha R}$$

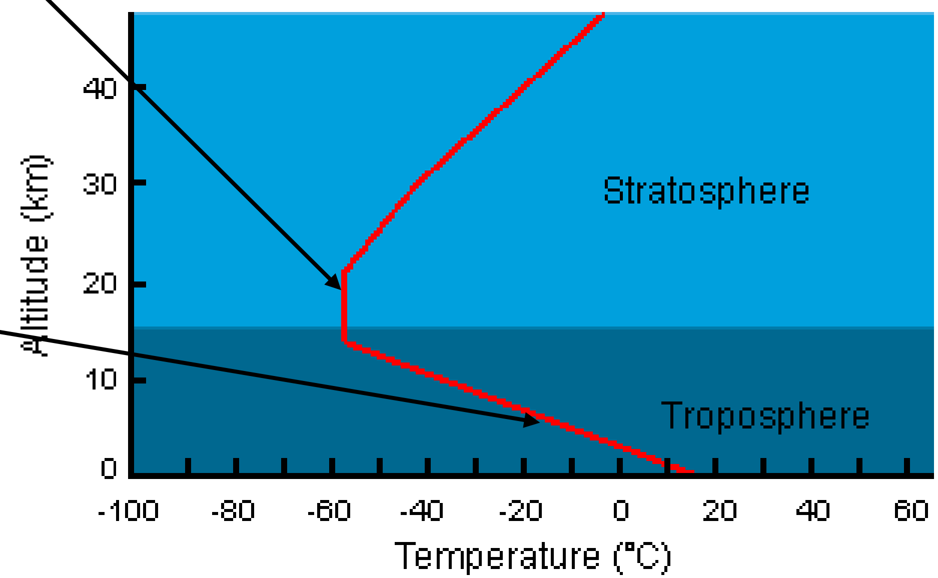
Atmospheric Equations

- Assume constant

$$p(z) = p_0 e^{-gM(z-z_0)/RT_0}$$

- Assume linear

$$p(z) = p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0} \right]^{gM/\alpha R}$$



Temperature variation with altitude for the U.S. standard atmosphere

Compressible Isentropic

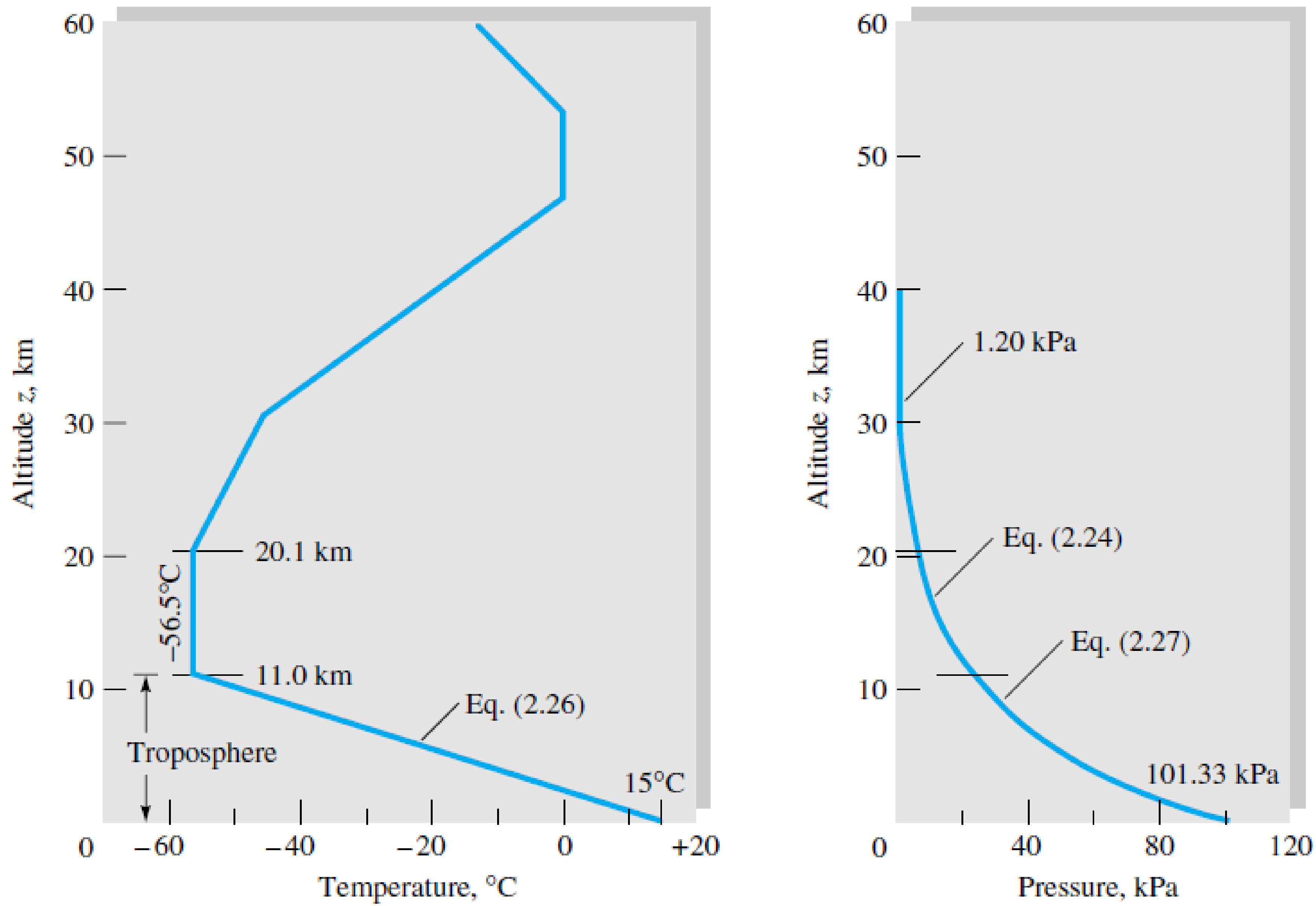
$$\frac{P}{\rho^{\gamma}} = constant = \frac{P_1}{\rho_1^{\gamma}}$$

$$\frac{T}{T_1} = \left(\frac{P}{P_1} \right)^{\gamma - 1/y}$$

$$\gamma = {C_p}/{C_v}$$

$$P_2 = P_1 \left[1 - \left(\frac{\gamma - 1}{\gamma} \right) \left(\frac{gM\Delta z}{RT_1} \right) \right]^{\gamma/\gamma - 1}$$

$$T_2 = T_1 \left[1 - \left(\frac{\gamma - 1}{\gamma} \right) \left(\frac{gM\Delta z}{RT_1} \right) \right]$$



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+20
Temperature, °C
1.20 kPa
Eq. (2.24)
Eq. (2.27)
101.33 kPa
40
80
120
Pressure, kPa

EXAMPLE 2.2

If sea-level pressure is 101,350 Pa, compute the standard pressure at an altitude of 5000 m, using (*a*) the exact formula and (*b*) an isothermal assumption at a standard sea-level temperature of 15°C. Is the isothermal approximation adequate?

Solution

Use absolute temperature in the exact formula, Eq. (2.27):

$$p = p_a\left[1 - \frac{(0.00650 \text{ K/m})(5000 \text{ m})}{288.16 \text{ K}}\right]^{5.26} = (101{,}350 \text{ Pa})(0.8872)^{5.26}$$

$$= 101{,}350(0.52388) = 54{,}000 \text{ Pa} \qquad \textit{Ans. (a)}$$

This is the standard-pressure result given at $z = 5000$ m in Table A.6.

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If the atmosphere were isothermal at 288.16 K, Eq. (2.24) would apply:

$$p \approx p_a \exp\left(-\frac{gz}{RT}\right) = (101{,}350\ \text{Pa}) \exp\left\{-\frac{(9.807\ \text{m/s}^2)(5000\ \text{m})}{[287\ \text{m}^2/(\text{s}^2\cdot\text{K})](288.16\ \text{K})}\right\}$$

$$= (101{,}350\ \text{Pa}) \exp(-0.5929) \approx 60{,}100\ \text{Pa} \qquad \textit{Ans. (b)}$$

This is 11 percent higher than the exact result. The isothermal formula is inaccurate in the troposphere.

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$$p = p_a\left(1 - \frac{Bz}{T_0}\right)^{g/(RB)} \qquad \text{where } \frac{g}{RB} = 5.26 \text{ (air)} \quad (2.27)$$

$$\left(1 - \frac{Bz}{T_0}\right)^n = 1 - n\frac{Bz}{T_0} + \frac{n(n-1)}{2!}\left(\frac{Bz}{T_0}\right)^2 - \cdots \quad (2.28)$$

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$$p = p_a - \gamma_a z\left(1 - \frac{n-1}{2}\frac{Bz}{T_0} + \cdots\right) \qquad (2.29)$$

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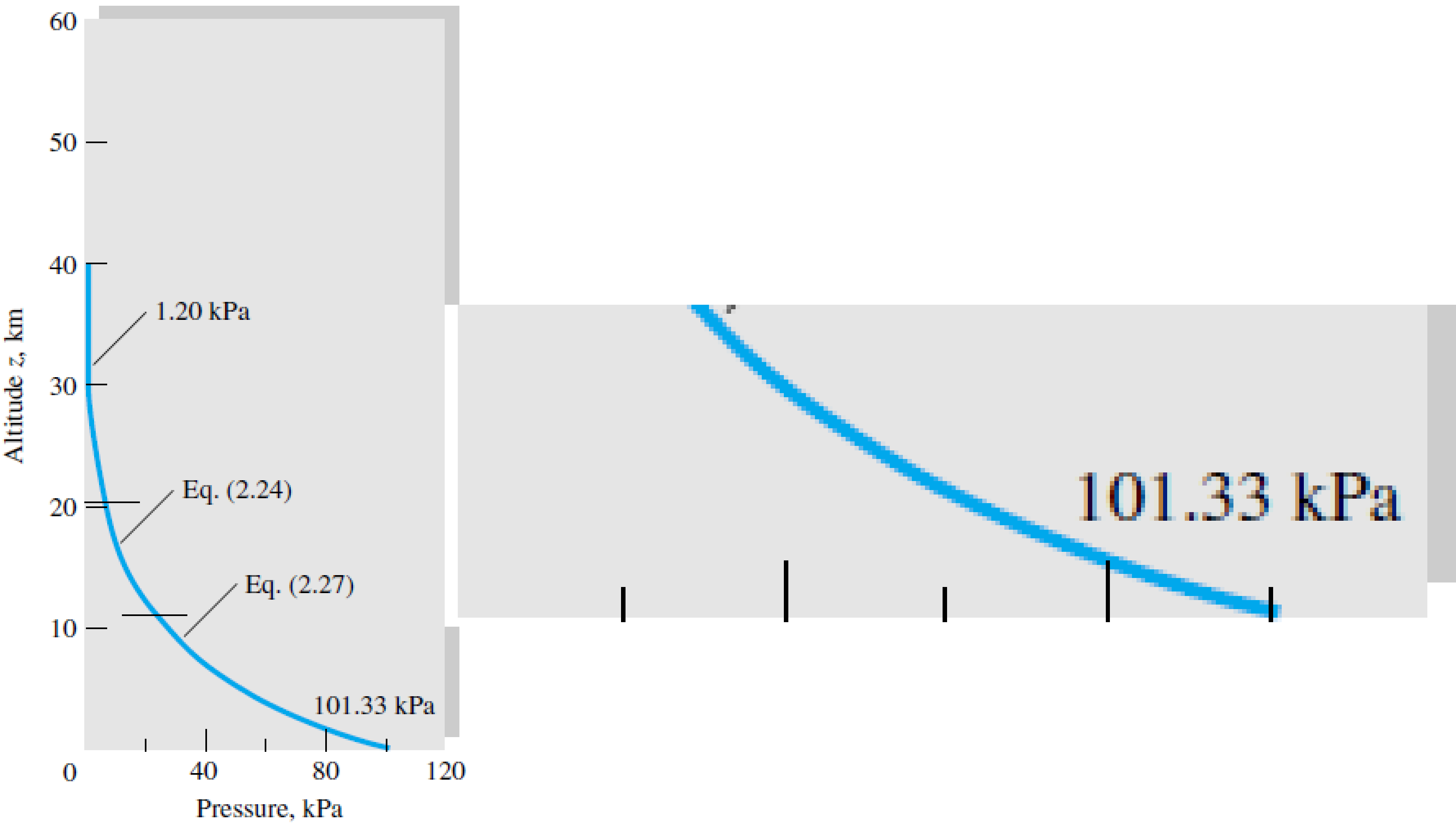
Thus the error in using the linear formula (2.21) is small if the second term in parentheses in (2.29) is small compared with unity. This is true if

$$z \ll \frac{2T_0}{(n-1)B} = 20{,}800 \text{ m} \tag{2.30}$$

We thus expect errors of less than 5 percent if z or δz is less than 1000 m.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1.20 kPa
Eq. (2.24)
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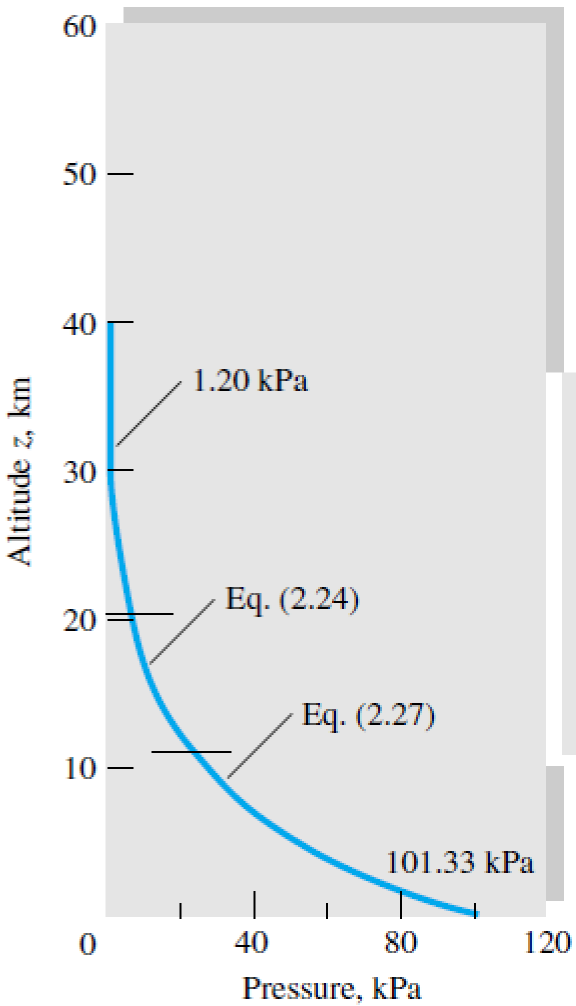
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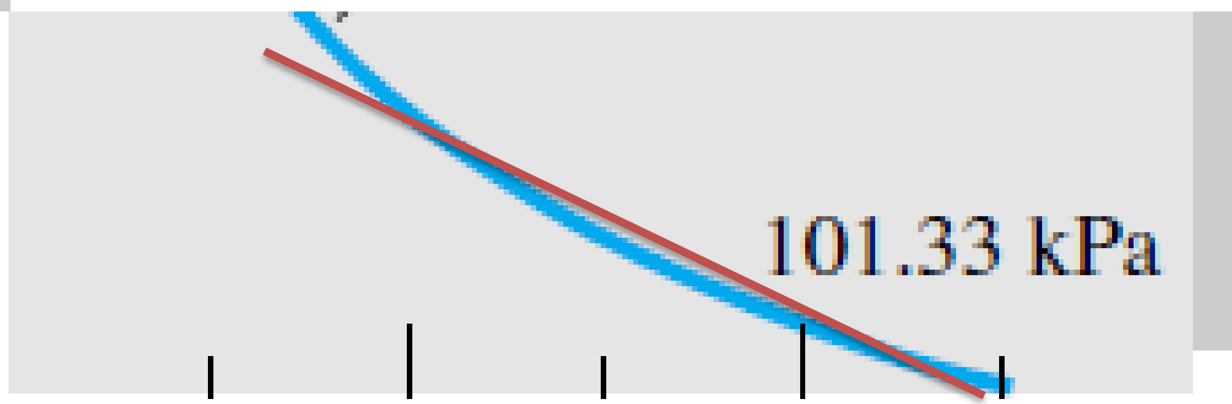
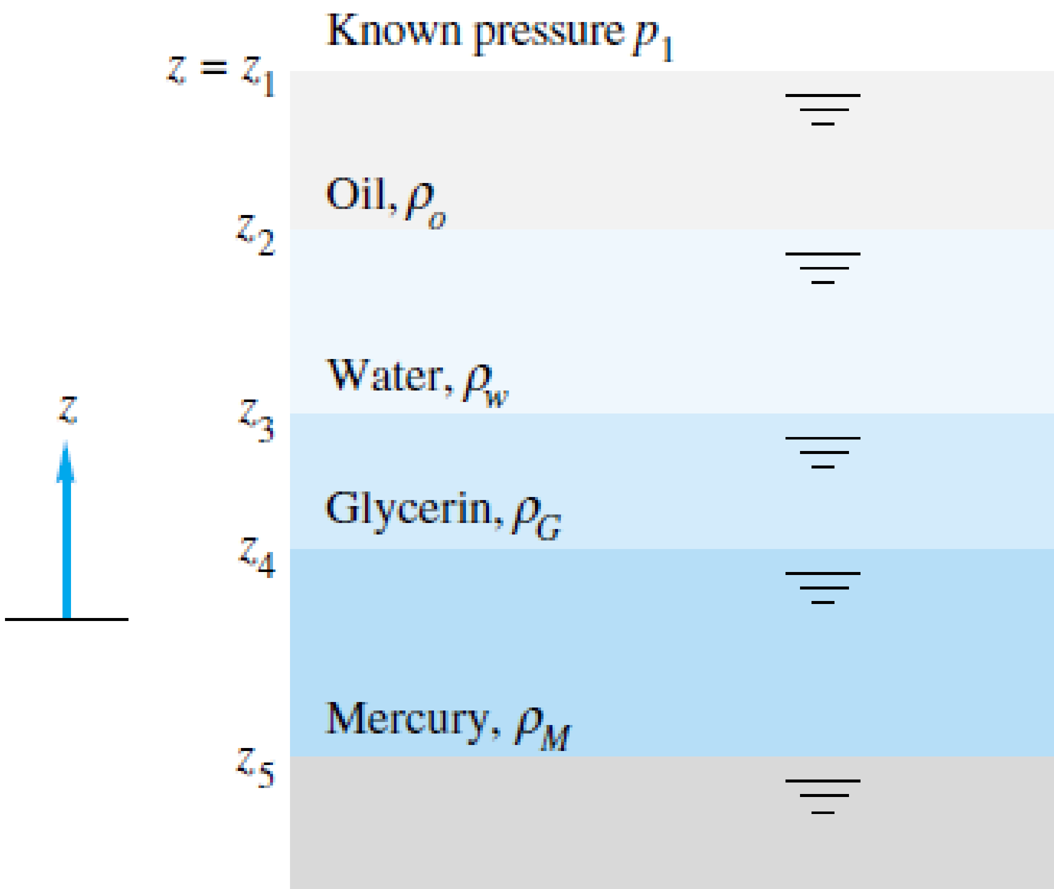

101.33 kPa

2.4 Application to Manometry

$$p_{down} = p^{up} + \gamma \left| \Delta z \right|$$



$$p_2 - p_1 = -\rho_o g(z_2 - z_1)$$

$$p_3 - p_2 = -\rho_w g(z_3 - z_2)$$

$$p_4 - p_3 = -\rho_G g(z_4 - z_3)$$

$$p_5 - p_4 = -\rho_M g(z_5 - z_4)$$

$$\text{Sum} = p_5 - p_1$$

2.4 Application to Manometry

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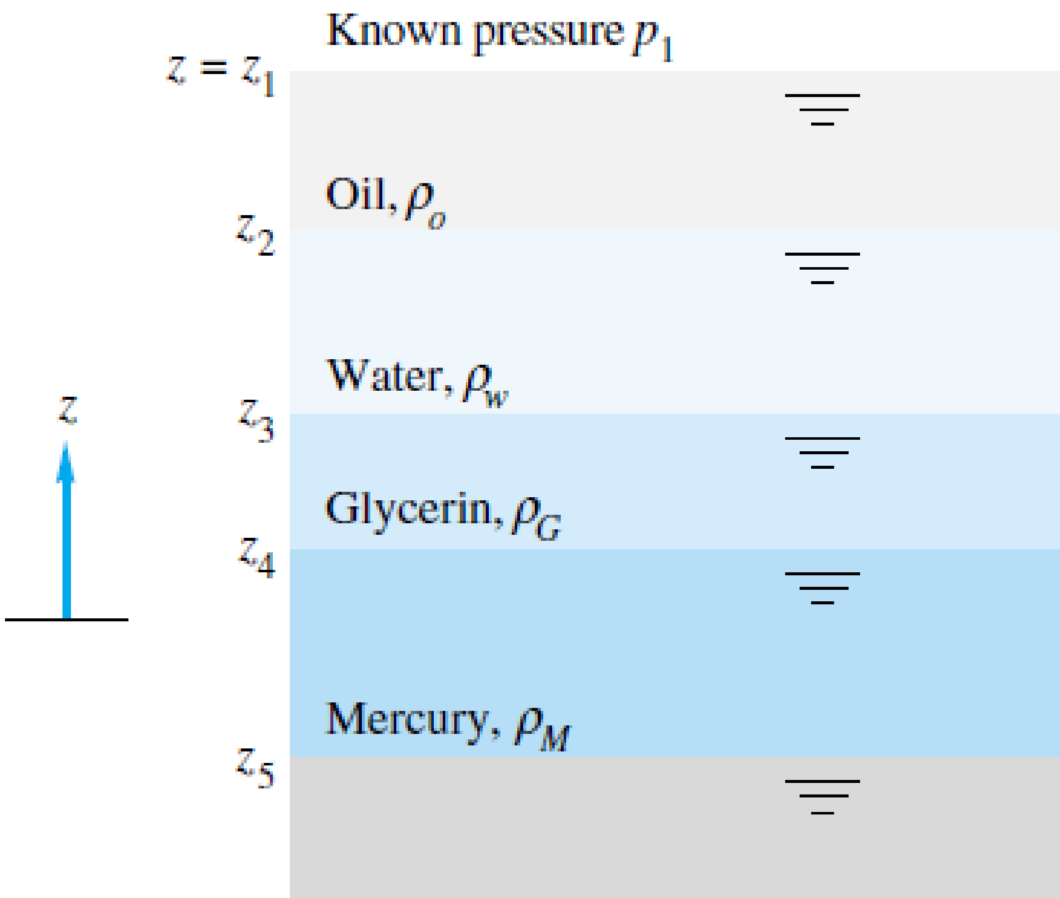


$$p_2 - p_1 = -\rho_o g(z_2 - z_1)$$

$$p_3 - p_2 = -\rho_w g(z_3 - z_2)$$

$$p_4 - p_3 = -\rho_G g(z_4 - z_3)$$

$$p_5 - p_4 = -\rho_M g(z_5 - z_4)$$

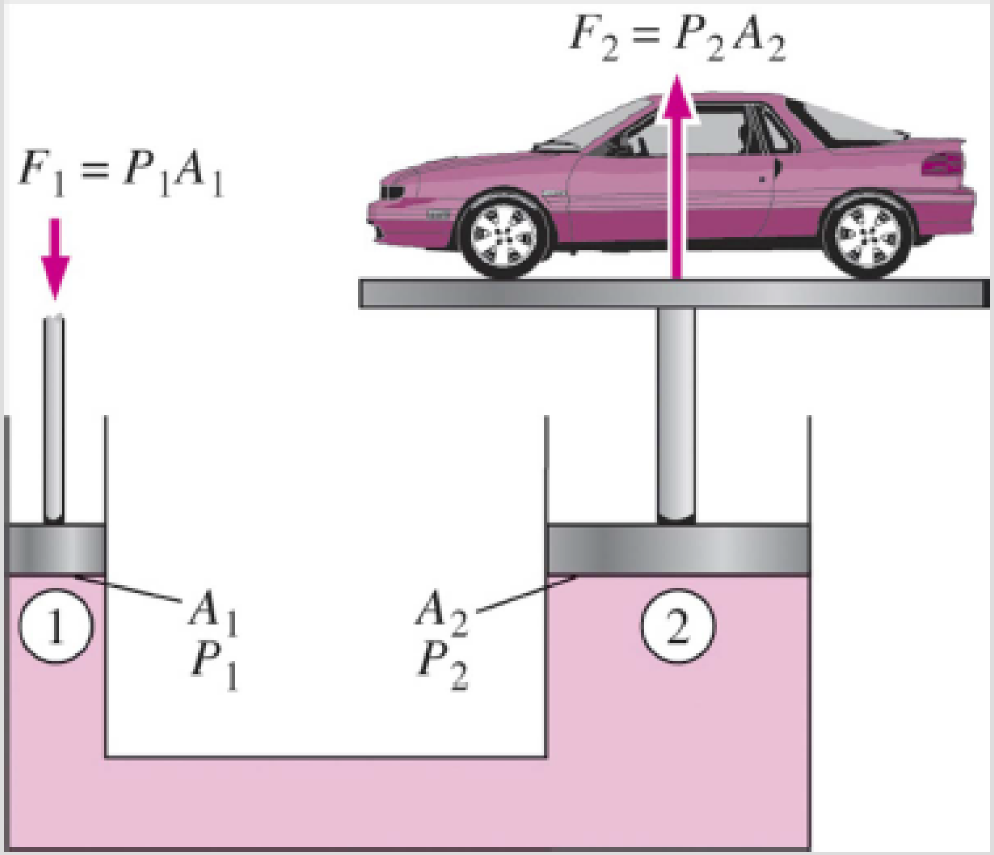
$$\text{Sum} = p_5 - p_1$$

$$p_5 = p_1 + \gamma_0|z_1 - z_2| + \gamma_w|z_2 - z_3| + \gamma_G|z_3 - z_4| + \gamma_M|z_4 - z_5|$$

Pascal's law: The pressure applied to a confined fluid increases the pressure throughout by the same amount.

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

The area ratio A_2/A_1 is called the *ideal mechanical advantage* of the hydraulic lift.



Lifting of a large weight by a small force by the application of Pascal's law.

Measurement of Pressure: Manometry

Manometry is a standard technique for measuring pressure using liquid columns in vertical or include tubes. The devices used in this manner are known as manometers.

The operation of three types of manometers will be discussed today:

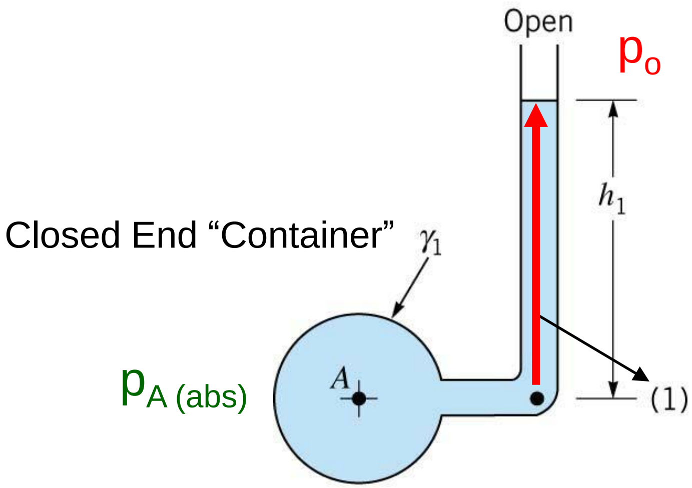
1) The Piezometer Tube
2) The U-Tube Manometer
3) The Inclined Tube Manometer

The fundamental equation for manometers since they involve columns of fluid at rest is the following:

$$p = \gamma h + p_0$$

h is positive moving downward, and negative moving upward, that is pressure in columns of fluid decrease with gains in height, and increase with gain in depth.

Measurement of Pressure: Piezometer Tube



Note: $p_A = p_1$ because they are at the same level

Moving from left to right: $p_{A(abs)} - \gamma_1 h_1 = p_o$

Rearranging: $p_A - p_o = \gamma_1 h_1$

Gage Pressure

Then in terms of gage pressure, the equation for a Piezometer Tube:

$$p_A = \gamma_1 h_1$$

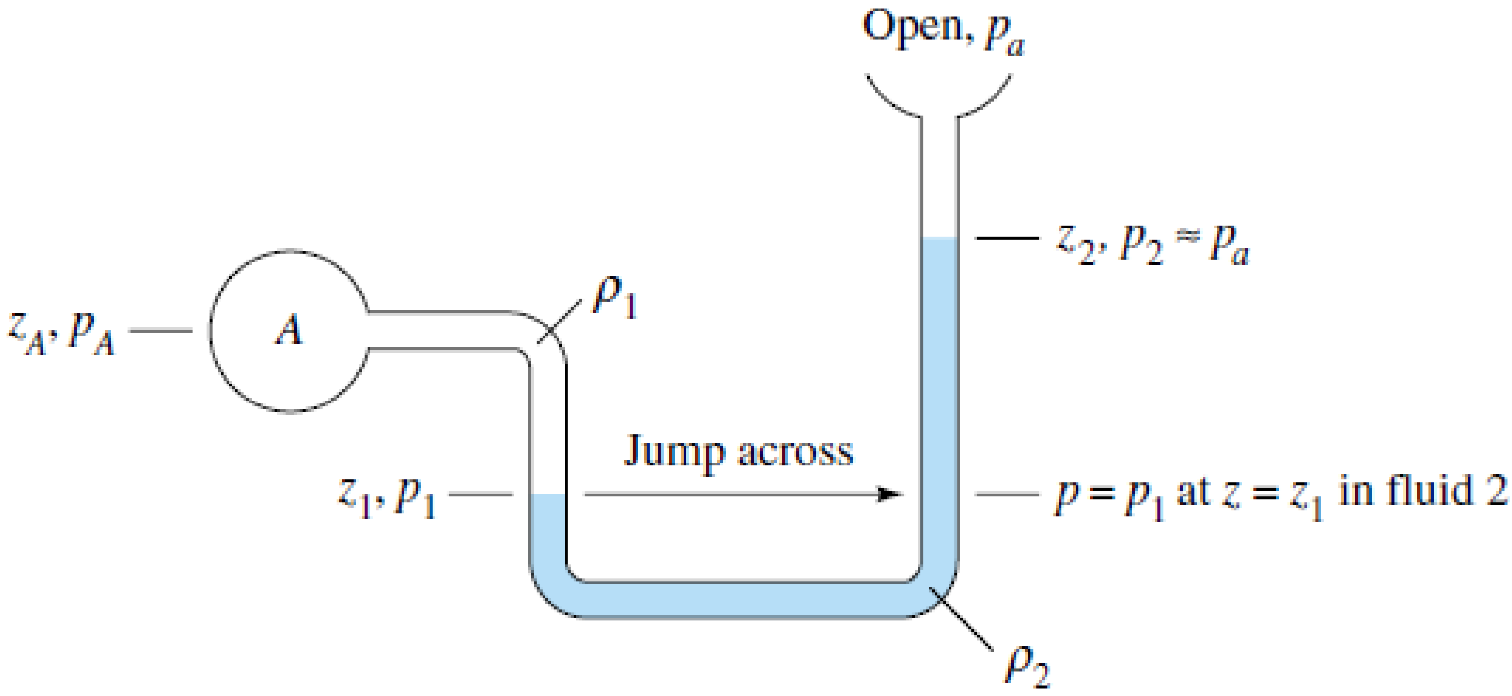
Measurement of Pressure: Piezometer Tube

Disadvantages:

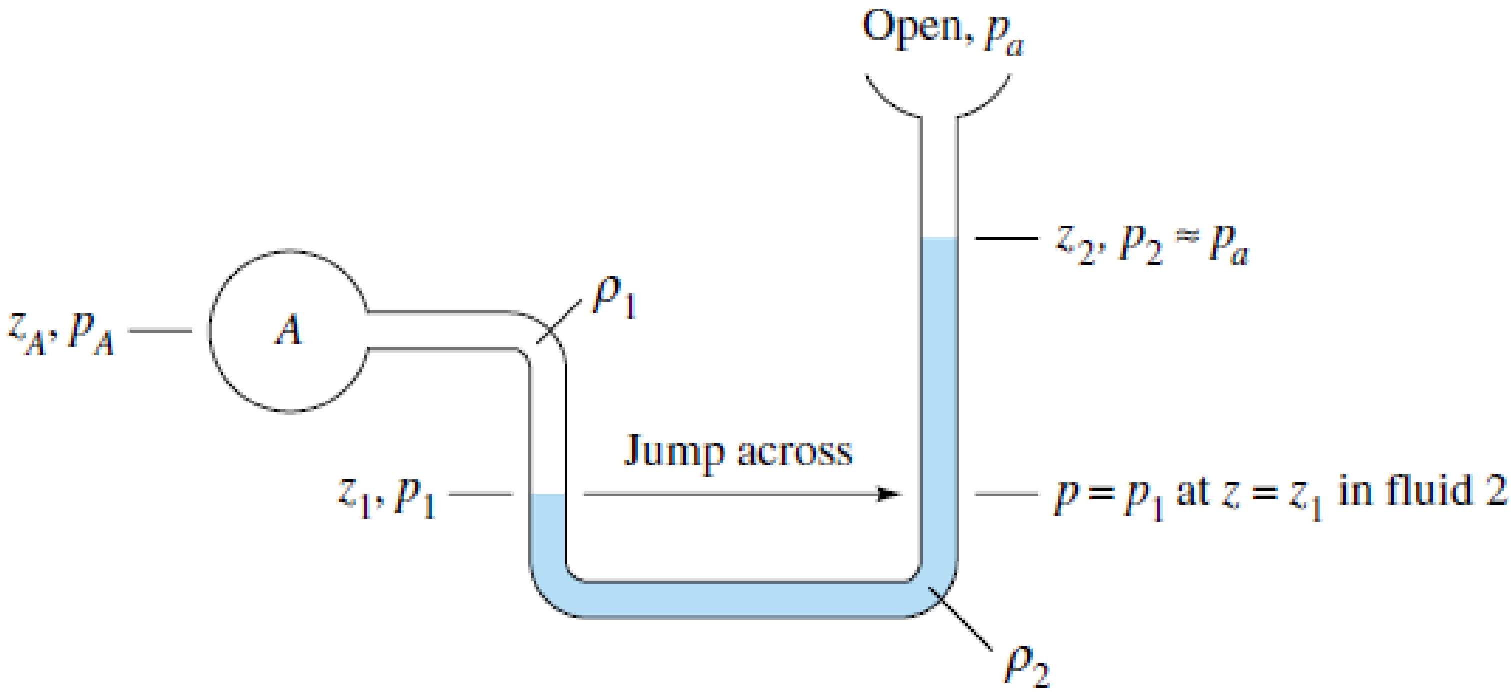
1)The pressure in the container has to be greater than atmospheric pressure.

2) Pressure must be relatively small to maintain a small column of fluid.

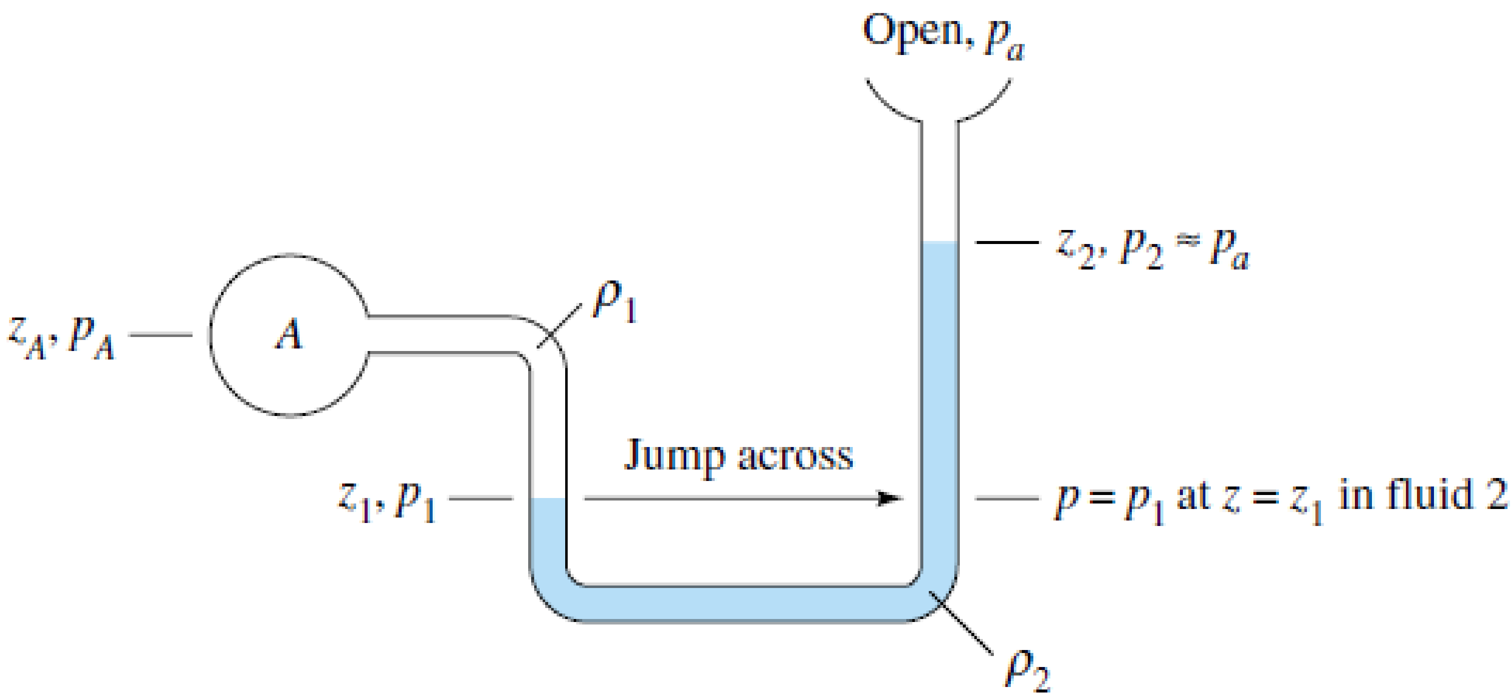
3) The measurement of pressure must be of a liquid.



Open, p_a
$z_2, p_2 \approx p_a$
ρ_1
z_A, p_A
A
Jump across
z_1, p_1
$p = p_1$ at $z = z_1$ in fluid 2
ρ_2



$$p_A + \gamma_1 |z_A - z_1| - \gamma_2 |z_1 - z_2| = p_2 \approx p_{\text{atm}}$$

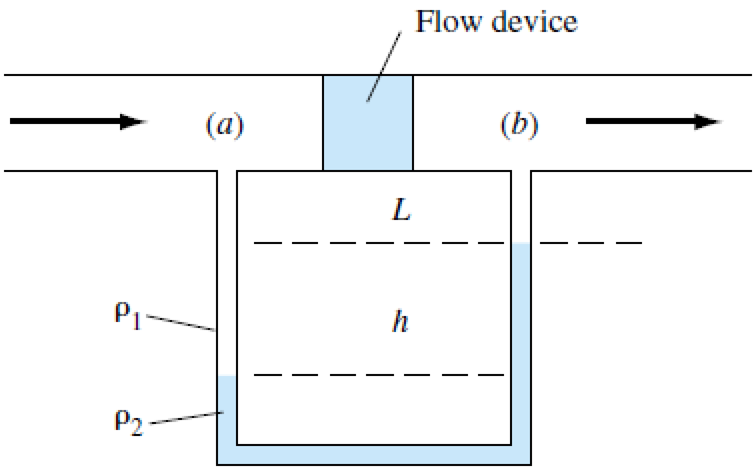


$$p_A + \gamma_1 |z_A - z_1| - \gamma_2 |z_1 - z_2| = p_2 \approx p_{atm}$$

Any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure.

EXAMPLE 2.3

The classic use of a manometer is when two U-tube legs are of equal length, as in Fig. E2.3, and the measurement involves a pressure difference across two horizontal points. The typical application is to measure pressure change across a flow device, as shown. Derive a formula for the pressure difference $p_a - p_b$ in terms of the system parameters in Fig. E2.3.



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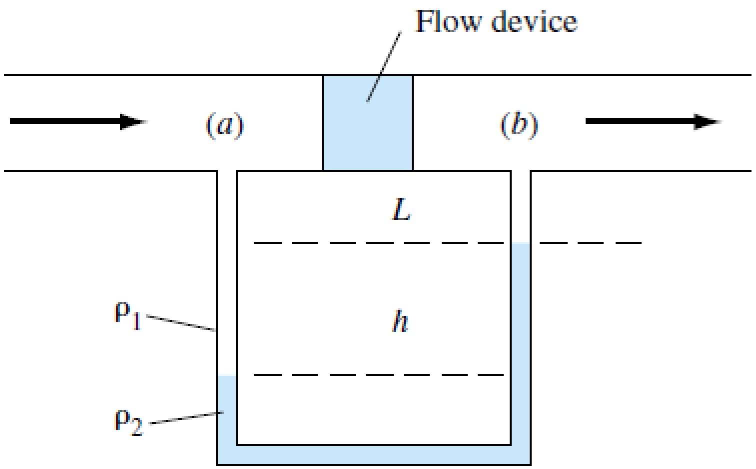
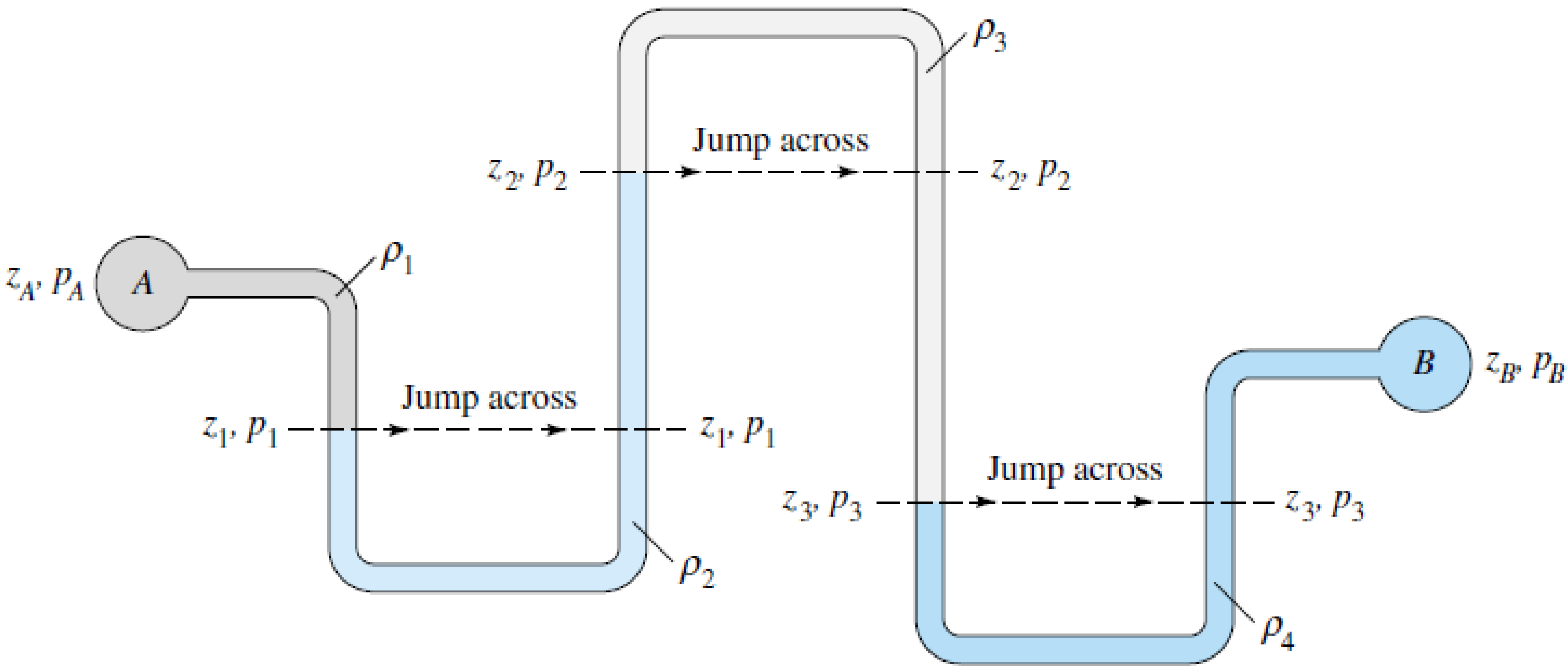
The classic use of a manometer is when two U-tube legs are of equal length, as in Fig. E2.3, and the measurement involves a pressure difference across two horizontal points. The typical ap-



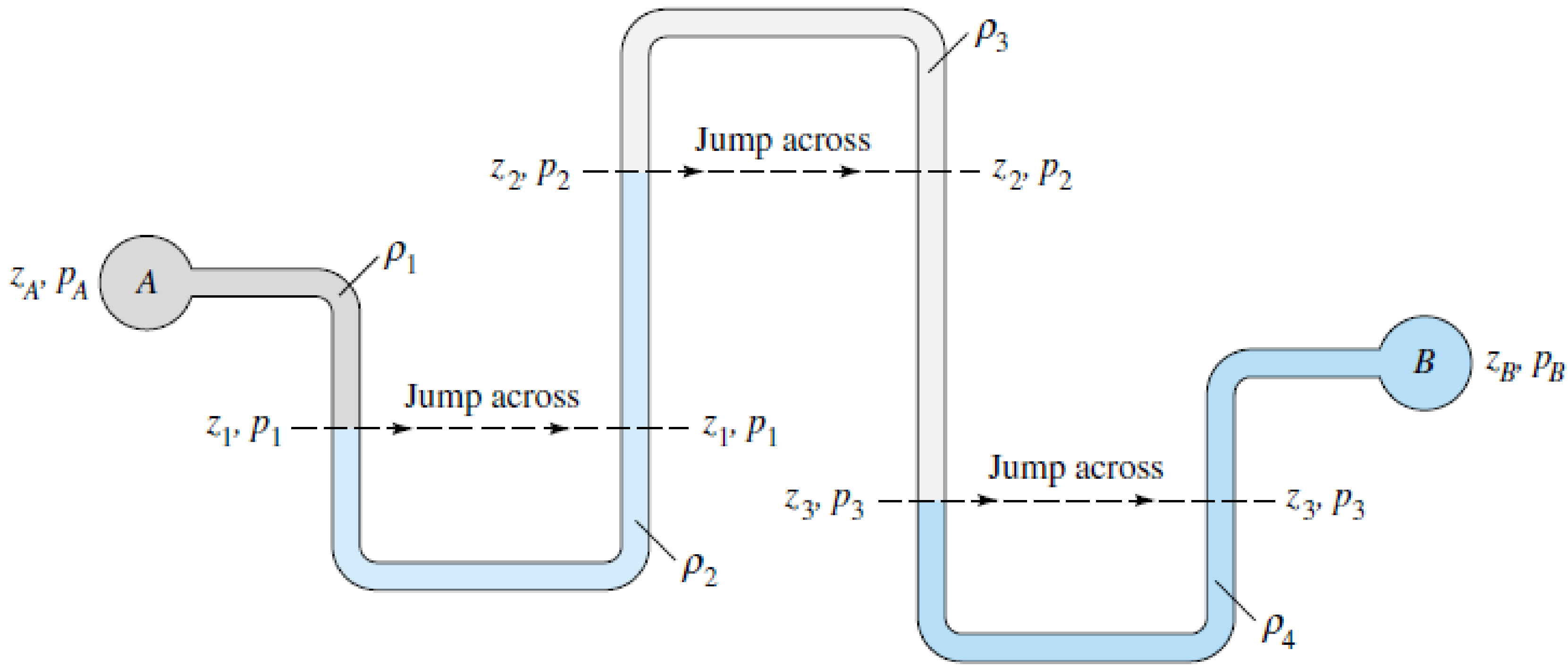
plication is to measure pressure change across a flow device, as shown. Derive a formula for the pressure difference $p_a - p_b$ in terms of the system parameters in Fig. E2.3.

$$p_a + \rho_1 g L + \rho_1 g h - \rho_2 g h - \rho_1 g L = p_b$$

$$p_a - p_b = (\rho_2 - \rho_1) g h$$



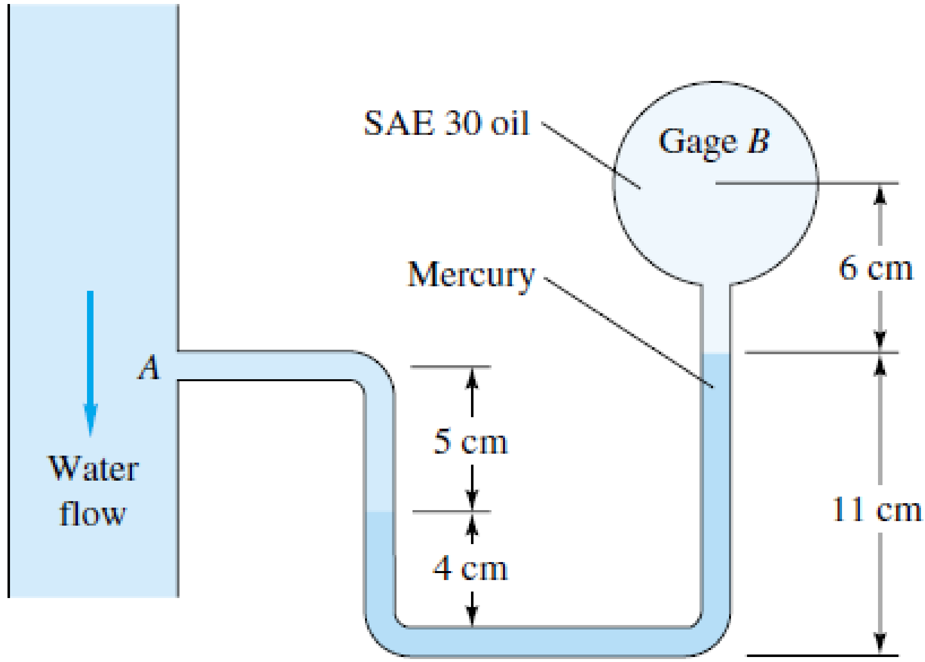
z_A, p_A
A
ρ_1
Jump across
z_1, p_1
z_1, p_1
z_2, p_2
Jump across
z_2, p_2
ρ_2
ρ_3
Jump across
z_3, p_3
z_3, p_3
ρ_4
B
z_B, p_B



$$p_A - p_B = (p_A - p_1) + (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_B)$$
$$= -\gamma_1(z_A - z_1) - \gamma_2(z_1 - z_2) - \gamma_3(z_2 - z_3) - \gamma_4(z_3 - z_B)$$

EXAMPLE 2.4

Pressure gage *B* is to measure the pressure at point *A* in a water flow. If the pressure at *B* is 87 kPa, estimate the pressure at *A*, in kPa. Assume all fluids are at 20°C. See Fig. E2.4.



Solution

First list the specific weights from Table 2.1 or Table A.3:

$$\gamma_{\text{water}} = 9790\ \text{N/m}^3 \qquad \gamma_{\text{mercury}} = 133{,}100\ \text{N/m}^3 \qquad \gamma_{\text{oil}} = 8720\ \text{N/m}^3$$

Now proceed from A to B, calculating the pressure change in each fluid and adding:

$$p_A - \gamma_W(\Delta z)_W - \gamma_M(\Delta z)_M - \gamma_O(\Delta z)_O = p_B$$

or $$p_A - (9790\ \text{N/m}^3)(-\,0.05\ \text{m}) - (133{,}100\ \text{N/m}^3)(0.07\ \text{m}) - (8720\ \text{N/m}^3)(0.06\ \text{m})$$

$$= p_A + 489.5\ \text{Pa} - 9317\ \text{Pa} - 523.2\ \text{Pa} = p_B = 87{,}000\ \text{Pa}$$

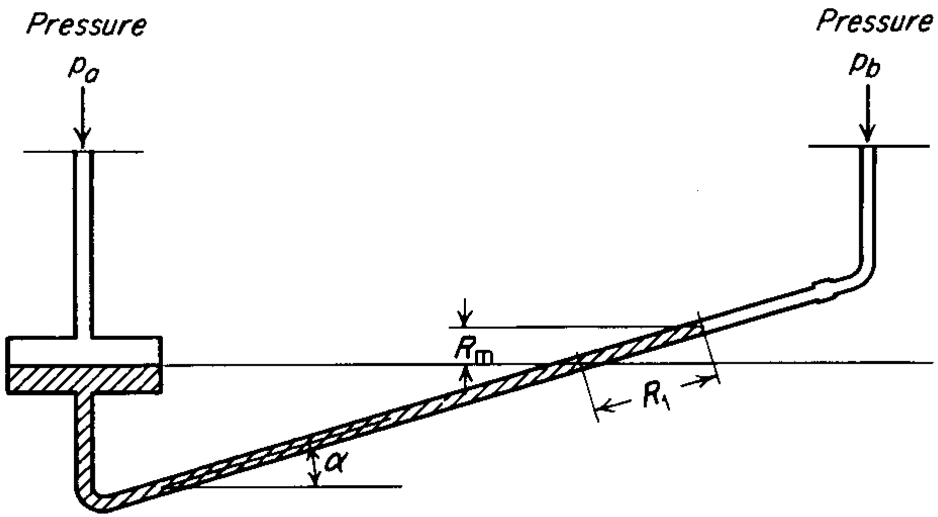
where we replace N/m^2 by its short name, Pa. The value $\Delta z_M = 0.07$ m is the net elevation change in the mercury (11 cm − 4 cm). Solving for the pressure at point A, we obtain

$$p_A = 96{,}351\ \text{Pa} = 96.4\ \text{kPa} \qquad \textit{Ans.}$$

The intermediate six-figure result of 96,351 Pa is utterly fatuous, since the measurements cannot be made that accurately.

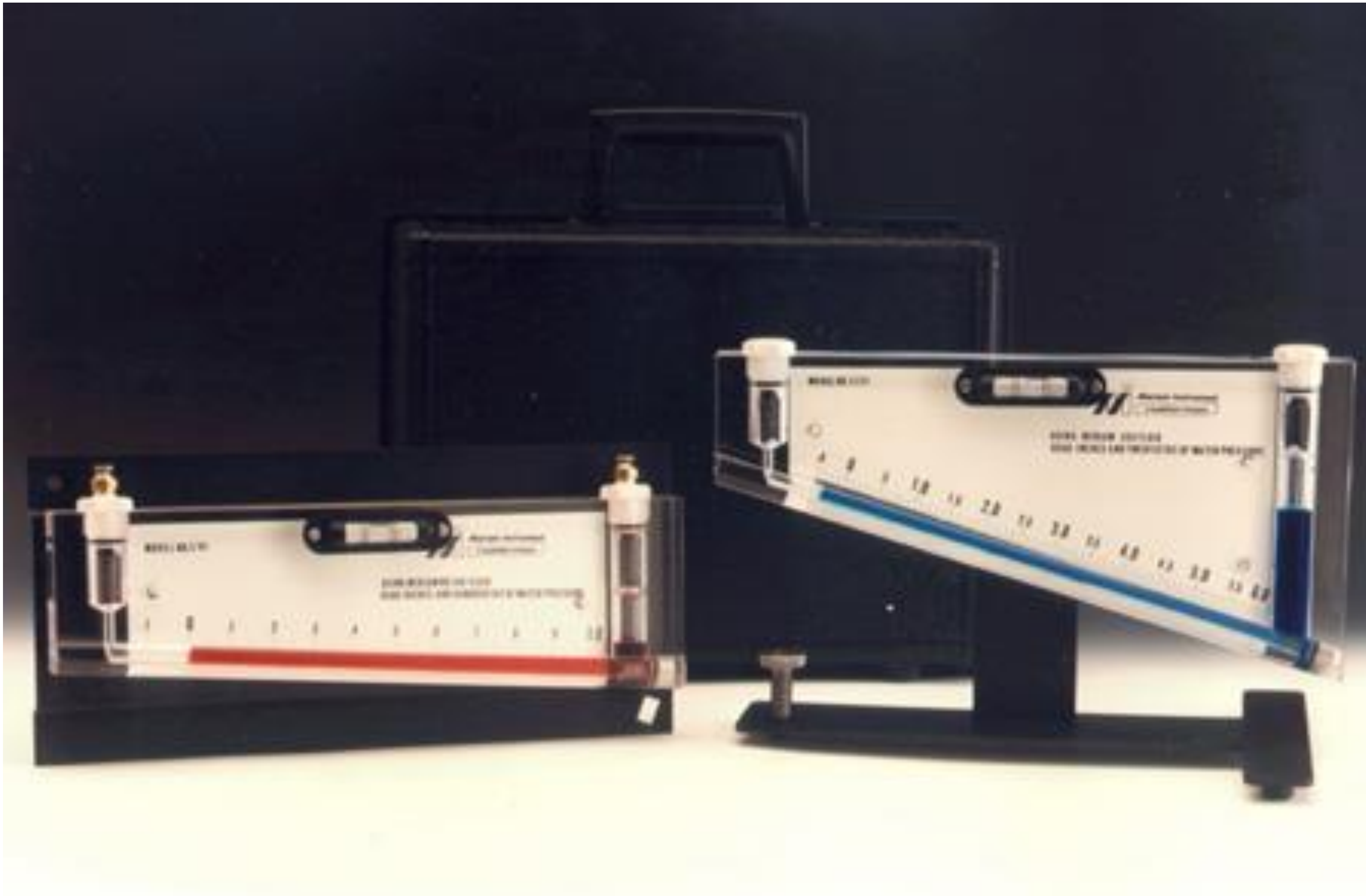
Inclined Manometer

- To measure small pressure differences need to magnify R_m some way.



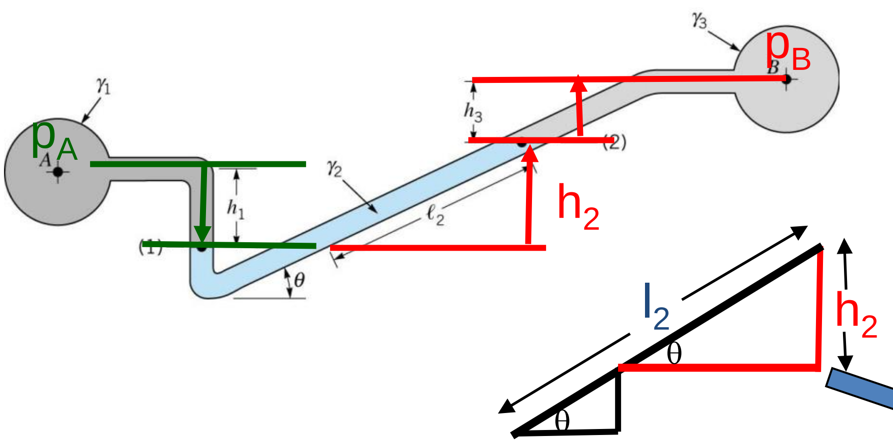
$$P_a - P_b = gR_1(\rho_a - \rho_b)\sin\alpha$$

Inclined Manometer



Measurement of Pressure: Inclined-Tube Manometer

This type of manometer is used to measure small pressure changes.



$$\sin\theta = \frac{h_2}{l_2} \Rightarrow h_2 = l_2 \sin\theta$$

Moving from left to right: $p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$

Substituting for h_2: $p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$

Rearranging to Obtain the Difference: $p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$

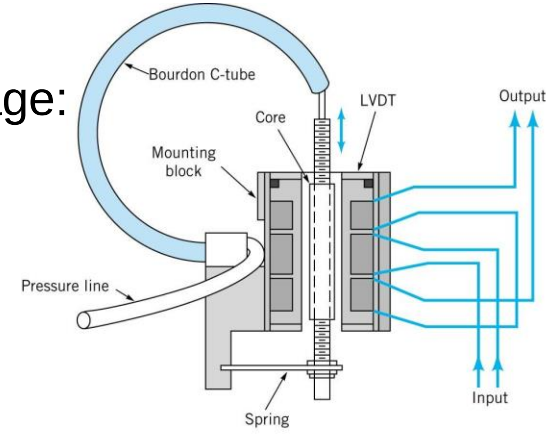
If the pressure difference is between gases: $p_A - p_B = \gamma_2 \ell_2 \sin\theta$

$$\ell_2 = \frac{p_A - p_B}{\gamma_2 \sin\theta}$$

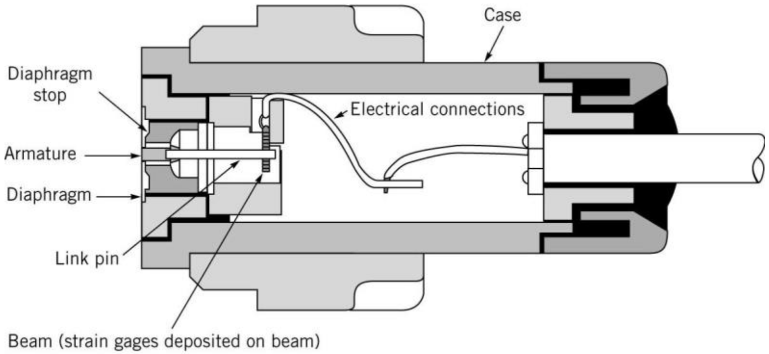
Thus, for the length of the tube we can measure a greater pressure differential.

Measurement of Pressure: Mechanical and Electrical Devices

Spring Bourdon Gage:



Diaphragm:



Pressure Measuring Devices - Pressure Transducer

- Pressure transducers generate an electrical signal as a function of the pressure they are exposed to.
- They work on many different technologies, such as
  - Piezoresistive
  - Piezoelectric
  - Capacitive
  - Electromagnetic
  - Optical
  - Thermal
  - etc.

- They can be used to measure pressure fluctuations in time.
- Differential types can measure pressure differences.