Chapter 12
Compartment Models

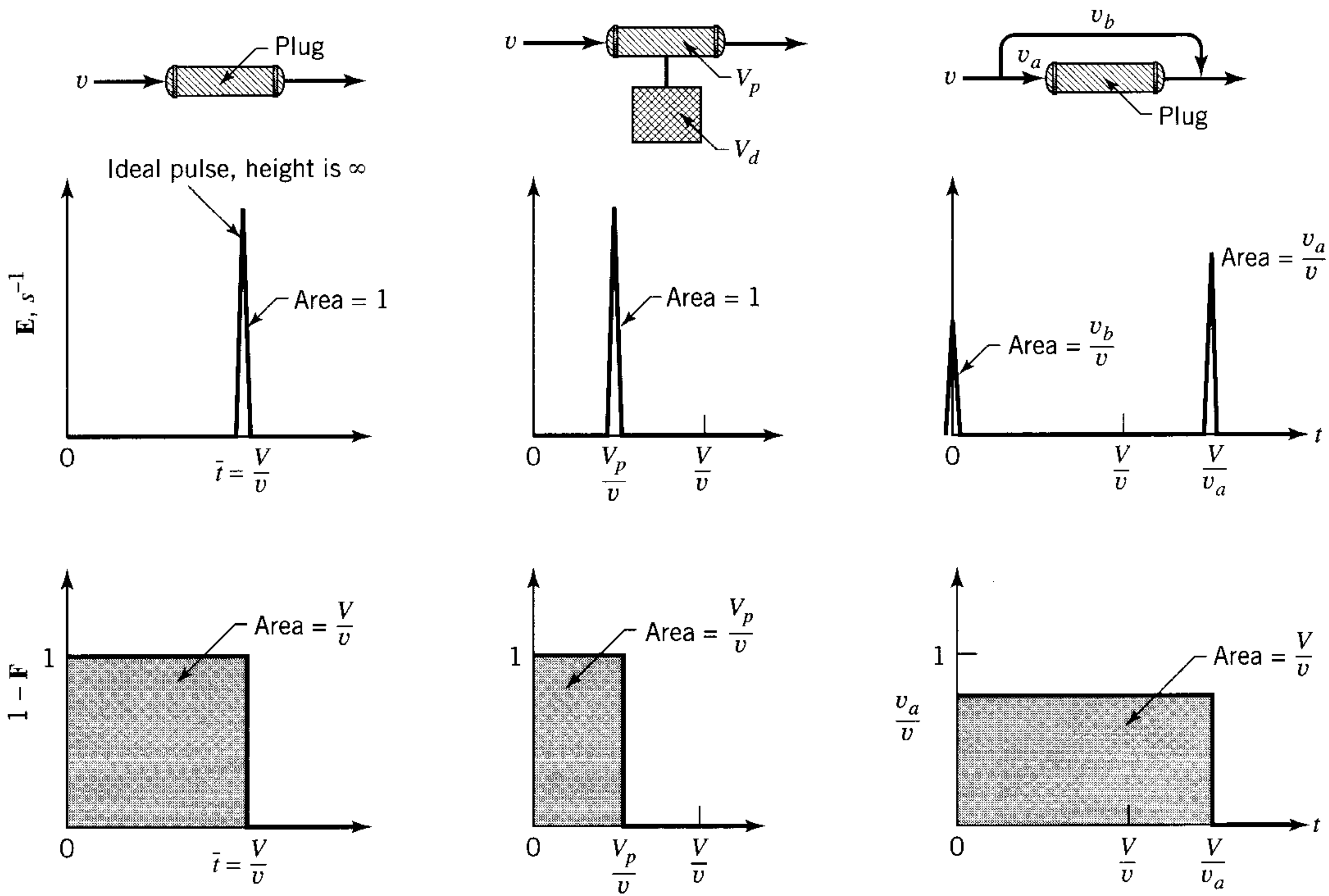
- In the compartment models we consider the vessel and the flow through it as follows:

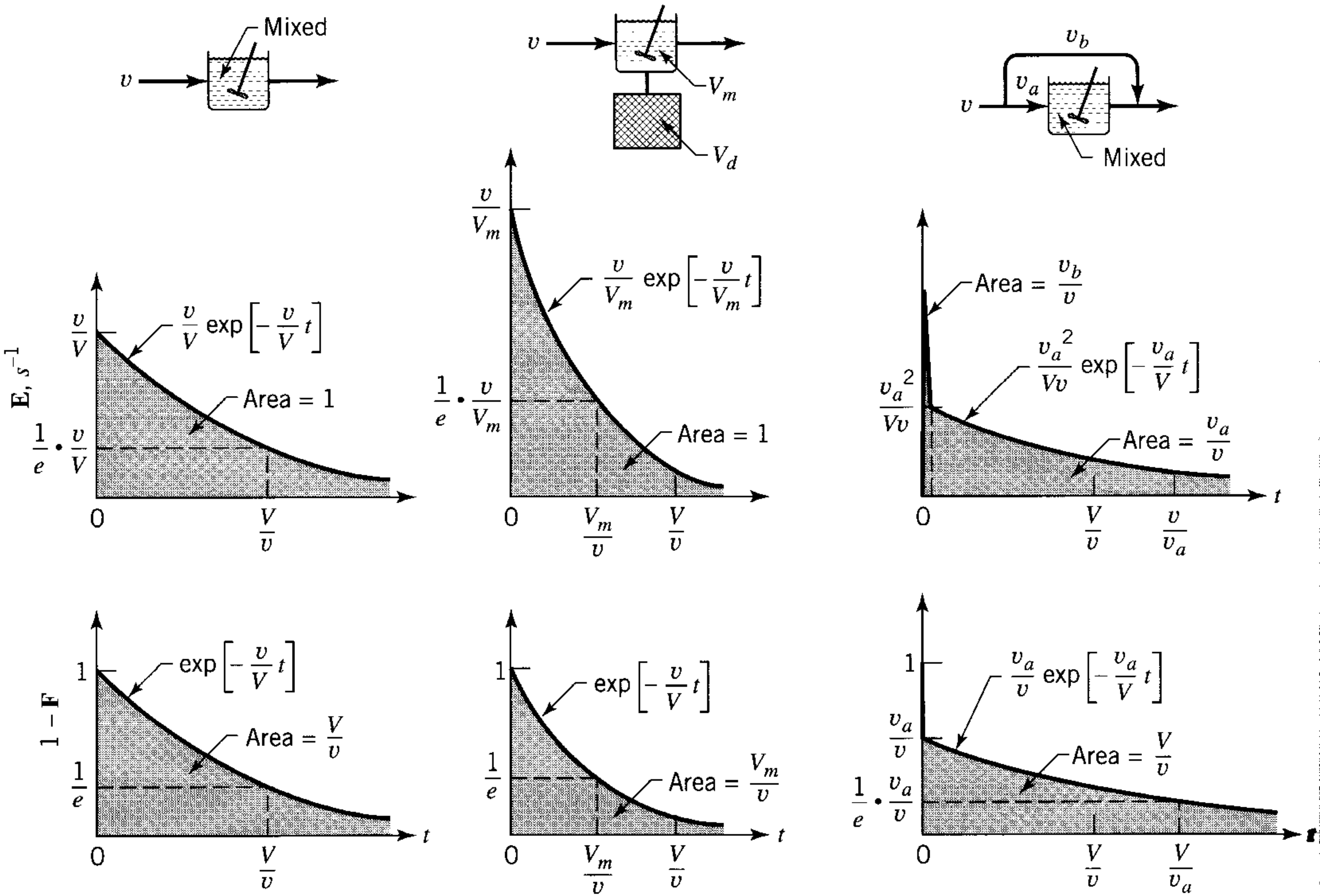
$$\begin{pmatrix} total \\ volume \\ V \end{pmatrix} \begin{cases} \left.\begin{matrix} V_p - plug\ flow\ region \\ V_m - mixed\ flow\ region \end{matrix}\right\} V_a - active\ volume \\ V_d - dead\ or\ stagnant\ \ region\ within\ the\ vessel \end{cases}$$

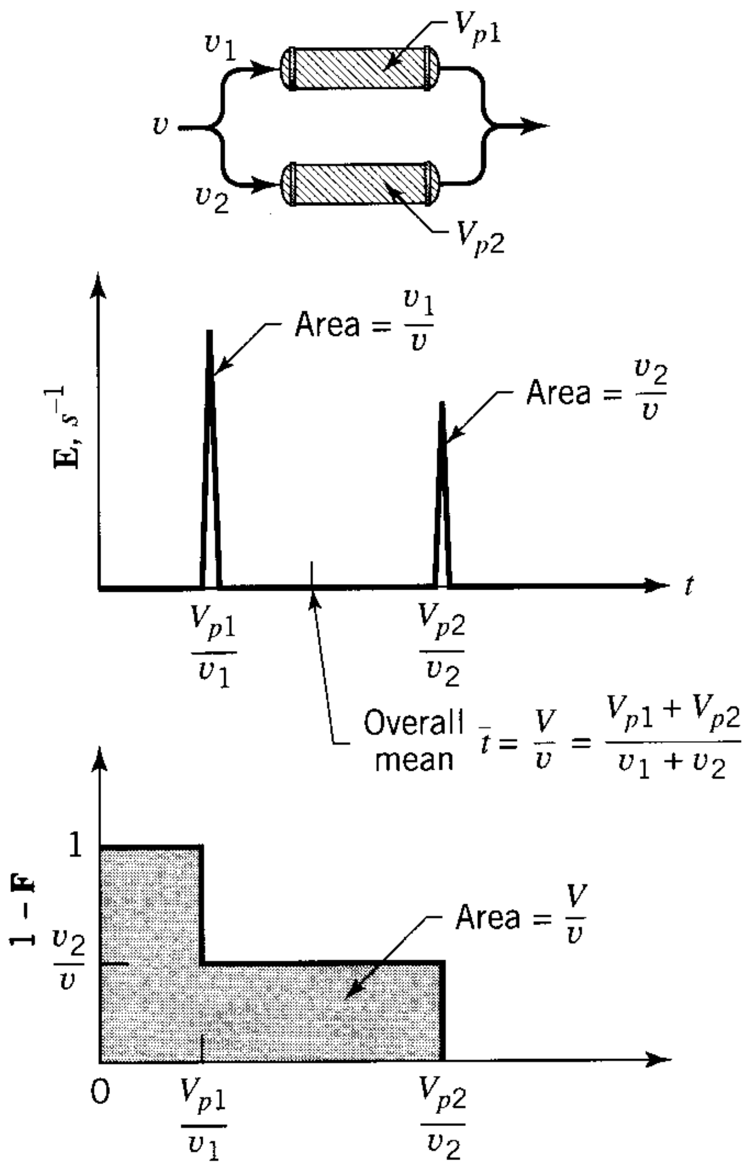
$$\begin{pmatrix} total \\ throughflow \\ v \end{pmatrix} \begin{cases} v_a - active\ flow, that\ through\ the \\ plug\ and\ mixed\ flow\ region \\ v_b - bypass\ flow\ region \\ v_r - recycle\ flow \end{cases}$$

- By comparing the E curve for the real vessel with the theoretical curves for various combinations of compartments and throughflow, we can find which model best fits the real vessel. Models of this kind are often a reasonable approximation to the real vessel.

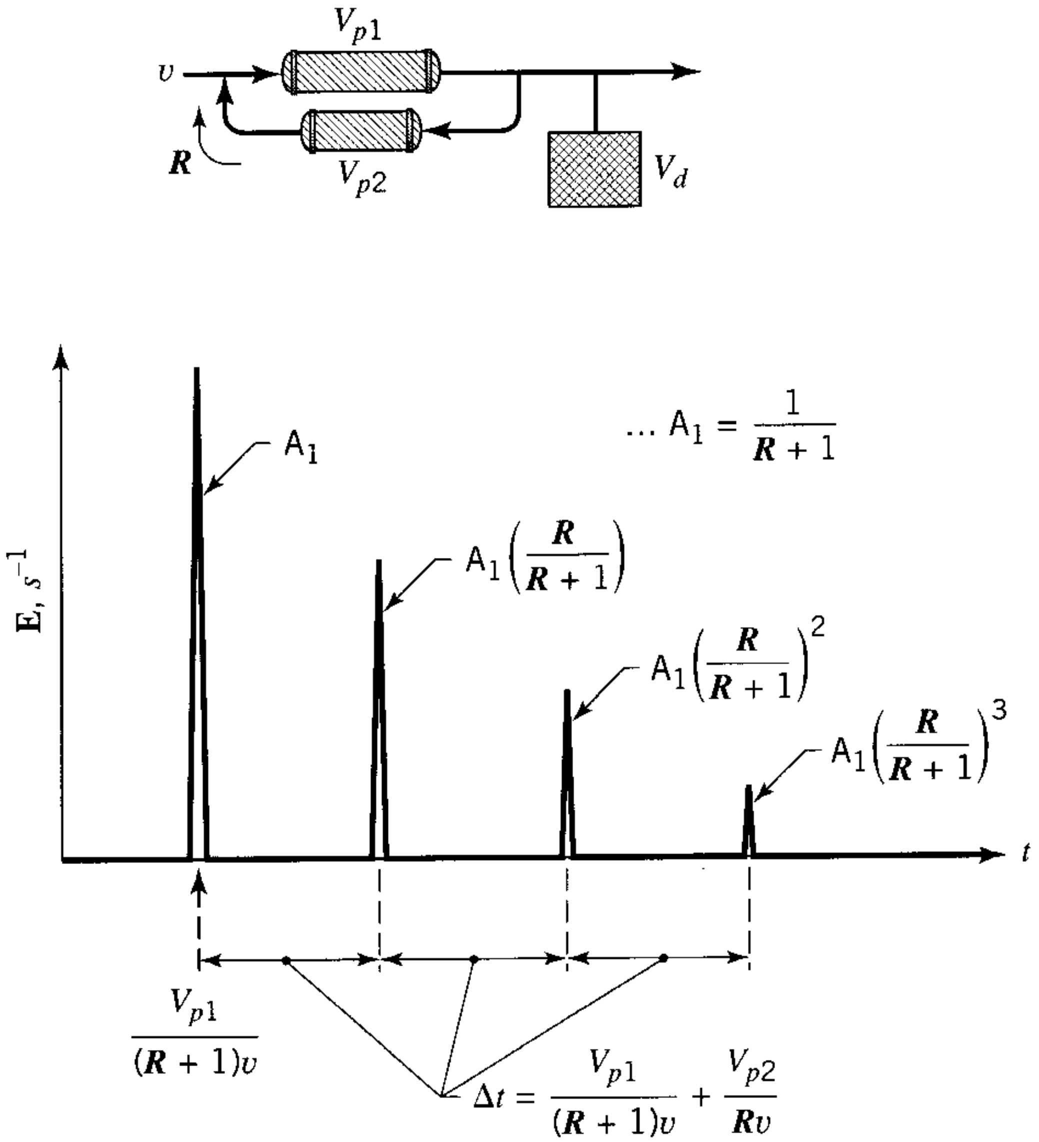
- We must know:
- M (kilograms of tracer introduced in the pulse)
- Both V and v
- The compartment model can be used for diagnosing reactor ills


Plug
v
V_p
V_d
v_b
v_a
Plug
Ideal pulse, height is ∞
E, s^-1
Area = 1
Area = 1
Area = v_a/v
Area = v_b/v
0
t̄ = V/v
V_p/v
V/v
V/v_a
t
1 − F
1
Area = V/v
Area = V_p/v
Area = V/v
v_a/v


Mixed
v
v
V_m
V_d
v_b
v_a
v
Mixed
E, s^-1
v/V
(v/V) exp[−(v/V) t]
Area = 1
(1/e)·(v/V)
0
V/v
v/V_m
(v/V_m) exp[−(v/V_m) t]
(1/e)·(v/V_m)
Area = 1
0
V_m/v
V/v
Area = v_b/v
(v_a^2/Vv) exp[−(v_a/V) t]
v_a^2/Vv
Area = v_a/v
0
V/v
v/v_a
t
1 − F
1
exp[−(v/V) t]
Area = V/v
1/e
0
V/v
t
1
exp[−(v/V) t]
1/e
Area = V_m/v
0
V_m/v
V/v
t
1
(v_a/v) exp[−(v_a/V) t]
v_a/v
Area = V/v
(1/e)·(v_a/v)
0
V/v
V/v_a
t



V_{p1}
v_1
v
v_2
V_{p2}
Area = $\frac{v_1}{v}$
Area = $\frac{v_2}{v}$
E, s^{-1}
t
$\frac{V_{p1}}{v_1}$
$\frac{V_{p2}}{v_2}$
Overall mean $\bar{t} = \frac{V}{v} = \frac{V_{p1} + V_{p2}}{v_1 + v_2}$
1 − F
1
$\frac{v_2}{v}$
Area = $\frac{V}{v}$
0



V_{p1}
v
R
V_{p2}
V_d
E, s^{-1}
$\dots A_1 = \frac{1}{R + 1}$
A_1
$A_1\left(\frac{R}{R+1}\right)$
$A_1\left(\frac{R}{R+1}\right)^2$
$A_1\left(\frac{R}{R+1}\right)^3$
t
$\frac{V_{p1}}{(R+1)v}$
$\Delta t = \frac{V_{p1}}{(R+1)v} + \frac{V_{p2}}{Rv}$

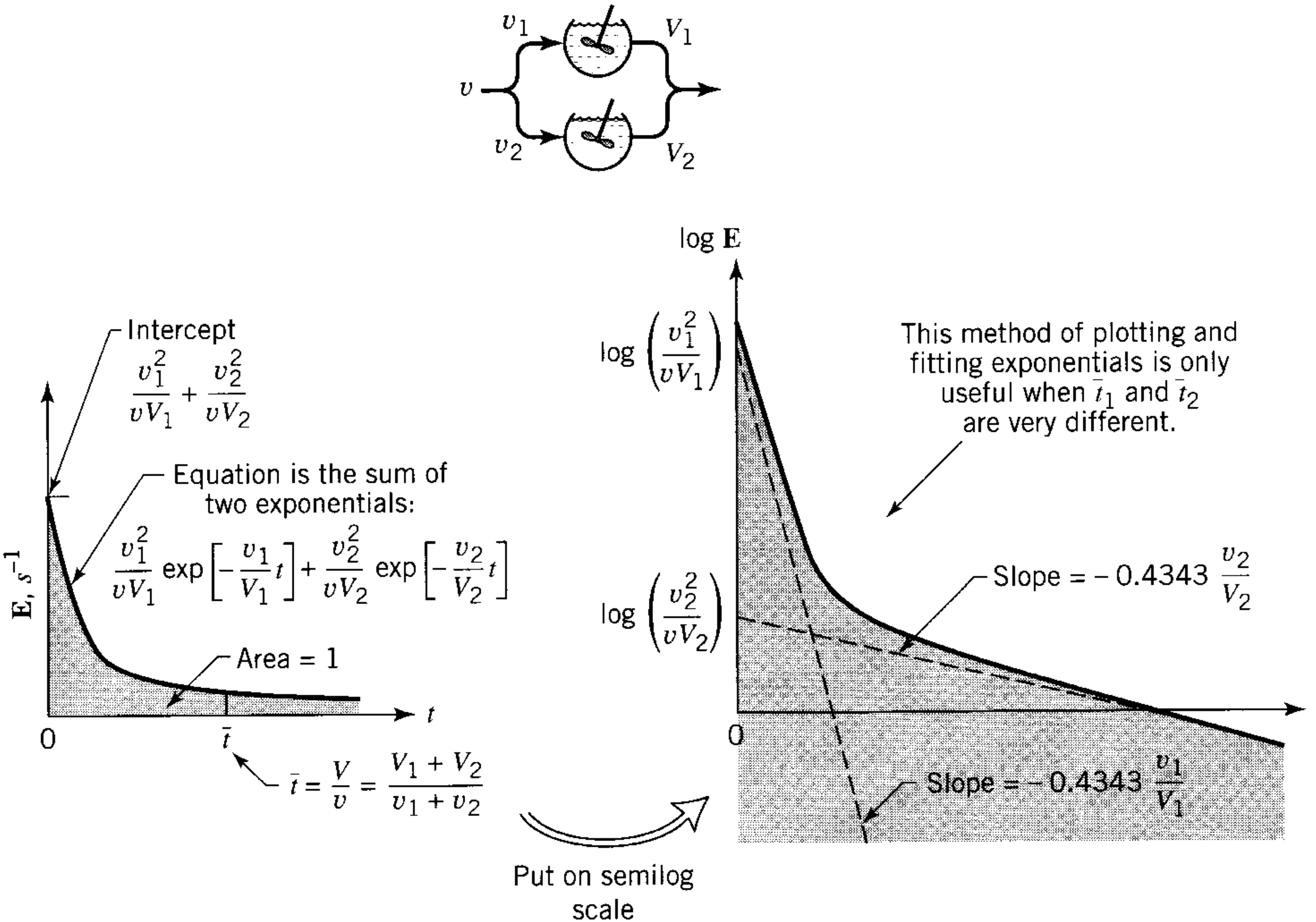
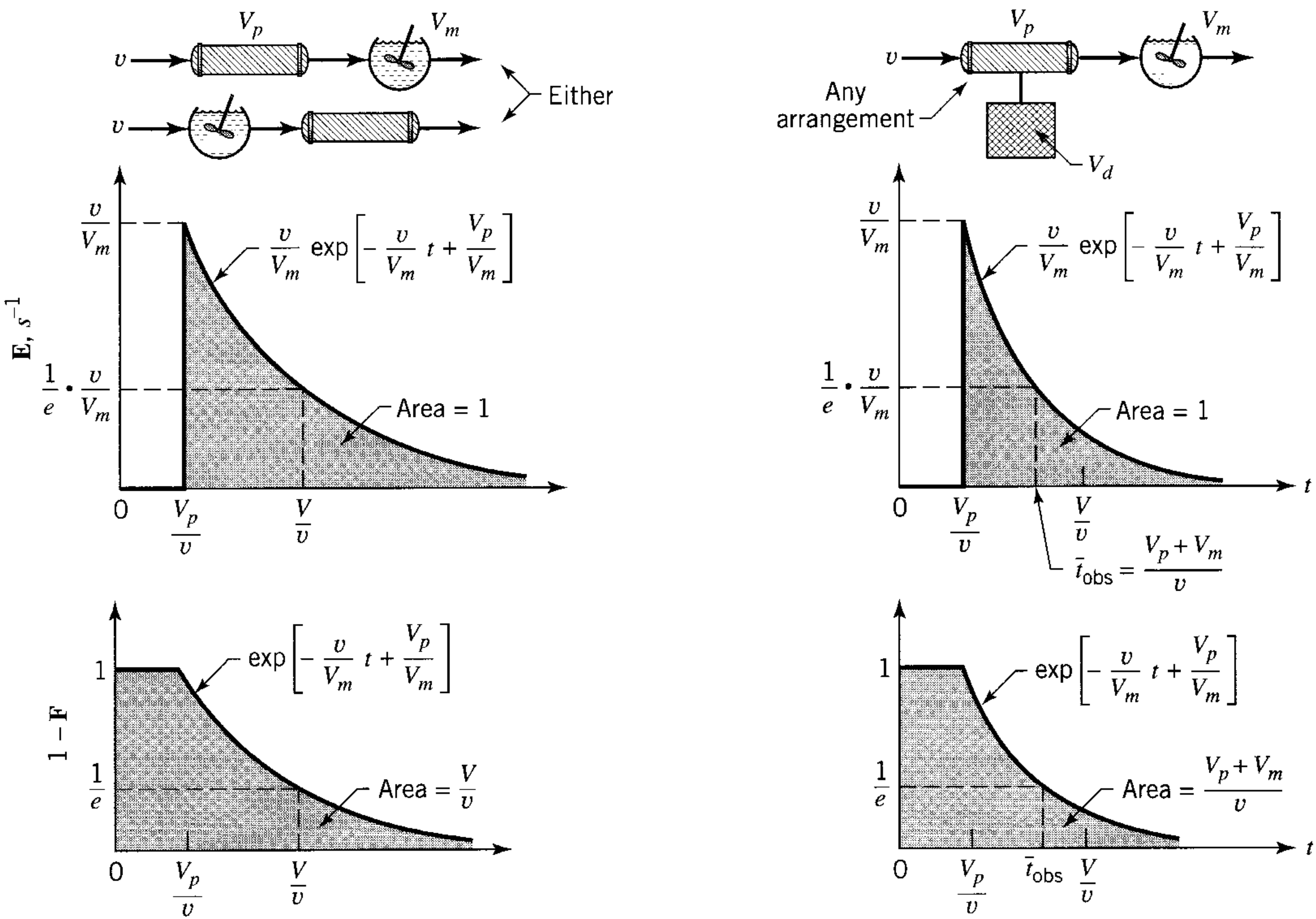
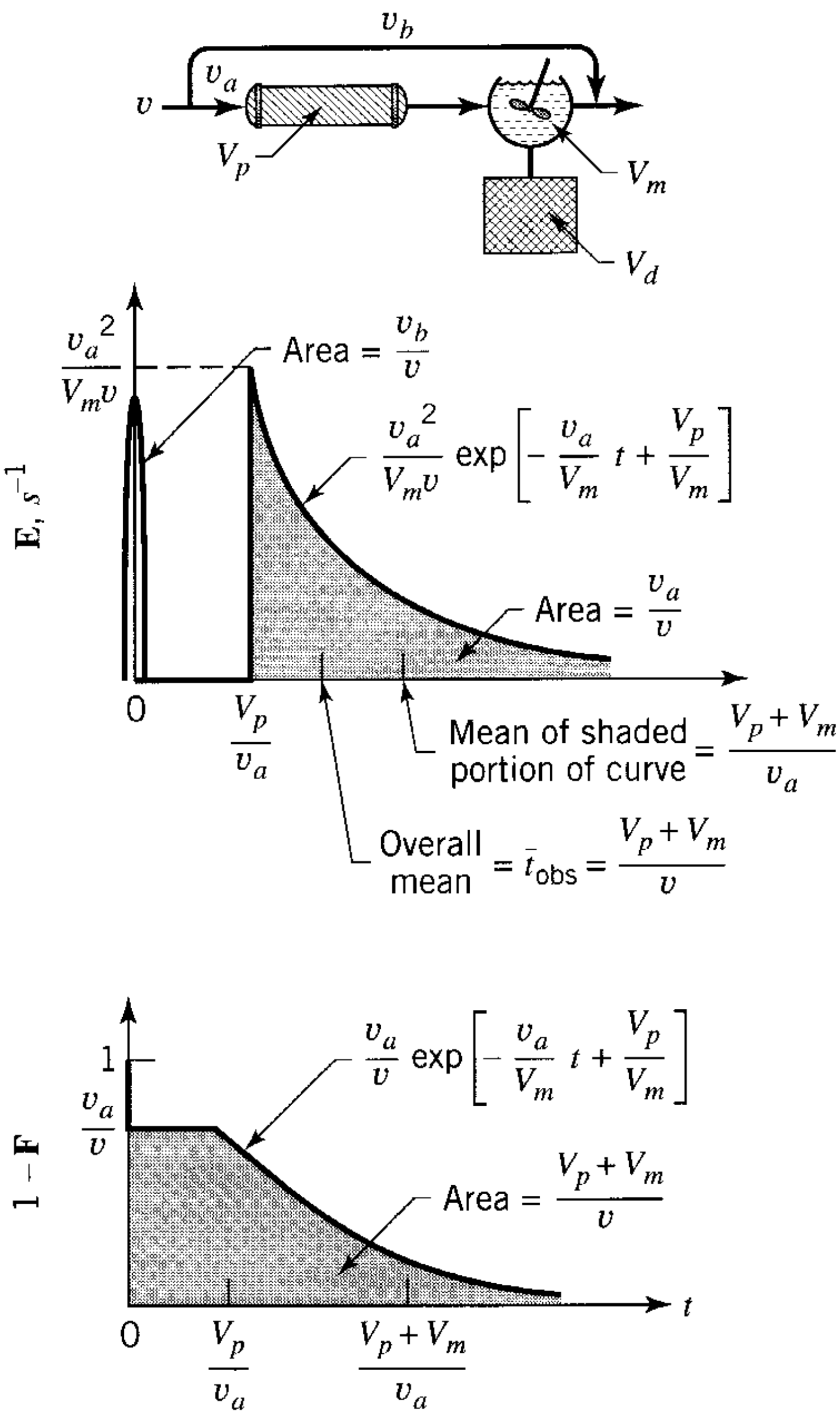


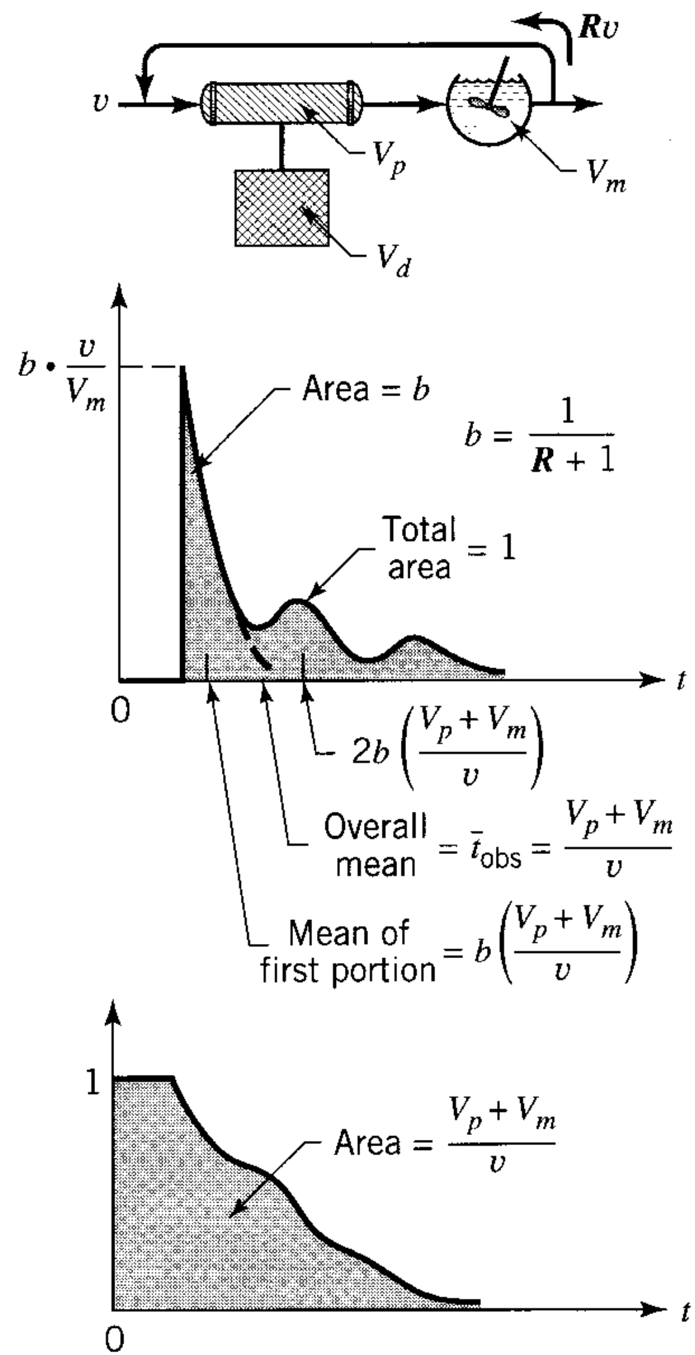
Figure 12.1 (*Continued*)



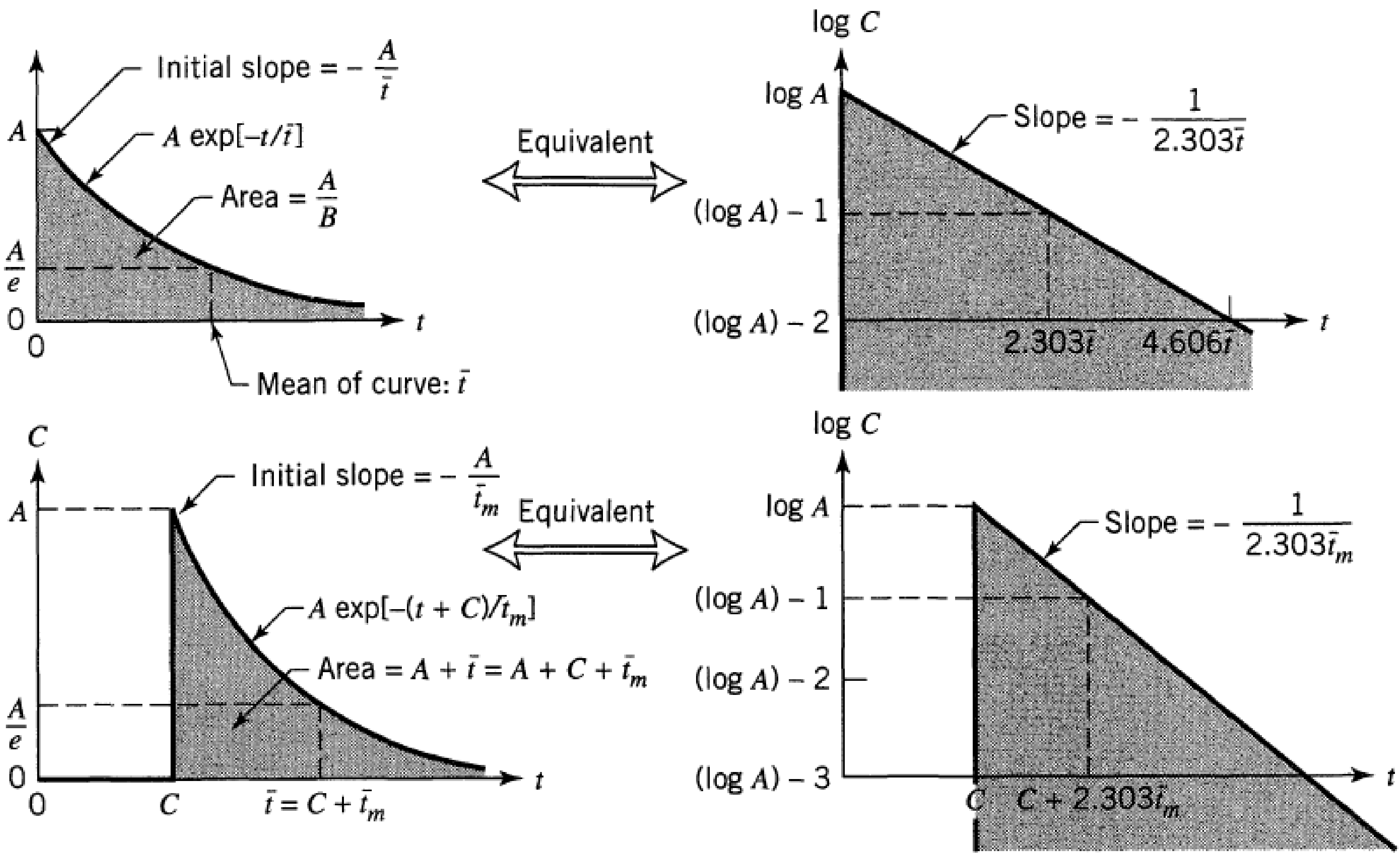
V_p
V_m
v
Either
Any arrangement
V_d
$\mathbf{E}, s^{-1}$
$\frac{v}{V_m}$
$\frac{1}{e} \cdot \frac{v}{V_m}$
$\frac{v}{V_m} \exp\left[-\frac{v}{V_m} t + \frac{V_p}{V_m}\right]$
Area = 1
0
$\frac{V_p}{v}$
$\frac{V}{v}$
t
$\bar{t}_{obs} = \frac{V_p + V_m}{v}$
$1 - \mathbf{F}$
1
$\frac{1}{e}$
$\exp\left[-\frac{v}{V_m} t + \frac{V_p}{V_m}\right]$
Area = $\frac{V}{v}$
Area = $\frac{V_p + V_m}{v}$
$\bar{t}_{obs}$



v_b
v_a
v
V_p
V_m
V_d
$\mathbf{E}, s^{-1}$
$\frac{v_a^2}{V_m v}$
Area = $\frac{v_b}{v}$
$\frac{v_a^2}{V_m v} \exp\left[-\frac{v_a}{V_m} t + \frac{V_p}{V_m}\right]$
Area = $\frac{v_a}{v}$
0
$\frac{V_p}{v_a}$
Mean of shaded portion of curve = $\frac{V_p + V_m}{v_a}$
Overall mean = $\bar{t}_{obs} = \frac{V_p + V_m}{v}$
$1 - \mathbf{F}$
1
$\frac{v_a}{v}$
$\frac{v_a}{v} \exp\left[-\frac{v_a}{V_m} t + \frac{V_p}{V_m}\right]$
Area = $\frac{V_p + V_m}{v}$
0
$\frac{V_p}{v_a}$
$\frac{V_p + V_m}{v_a}$
t



Rv
v
V_p
V_m
V_d
$b \cdot \frac{v}{V_m}$
Area = b
$b = \frac{1}{R + 1}$
Total area = 1
0
t
$2b\left(\frac{V_p + V_m}{v}\right)$
Overall mean = $\bar{t}_{obs} = \frac{V_p + V_m}{v}$
Mean of first portion = $b\left(\frac{V_p + V_m}{v}\right)$
1
Area = $\frac{V_p + V_m}{v}$
0
t



Initial slope = − A/t̄
A
A exp[−t/t̄]
Area = A/B
A/e
0
0
t
Mean of curve: t̄
Equivalent
log C
log A
Slope = − 1/2.303t̄
(log A) − 1
(log A) − 2
2.303t̄
4.606t̄
t
C
Initial slope = − A/t̄m
A
A exp[−(t + C)/t̄m]
Area = A + t̄ = A + C + t̄m
A/e
0
0
C
t̄ = C + t̄m
t
Equivalent
log C
log A
Slope = − 1/2.303t̄m
(log A) − 1
(log A) − 2
(log A) − 3
C
C + 2.303t̄m
t

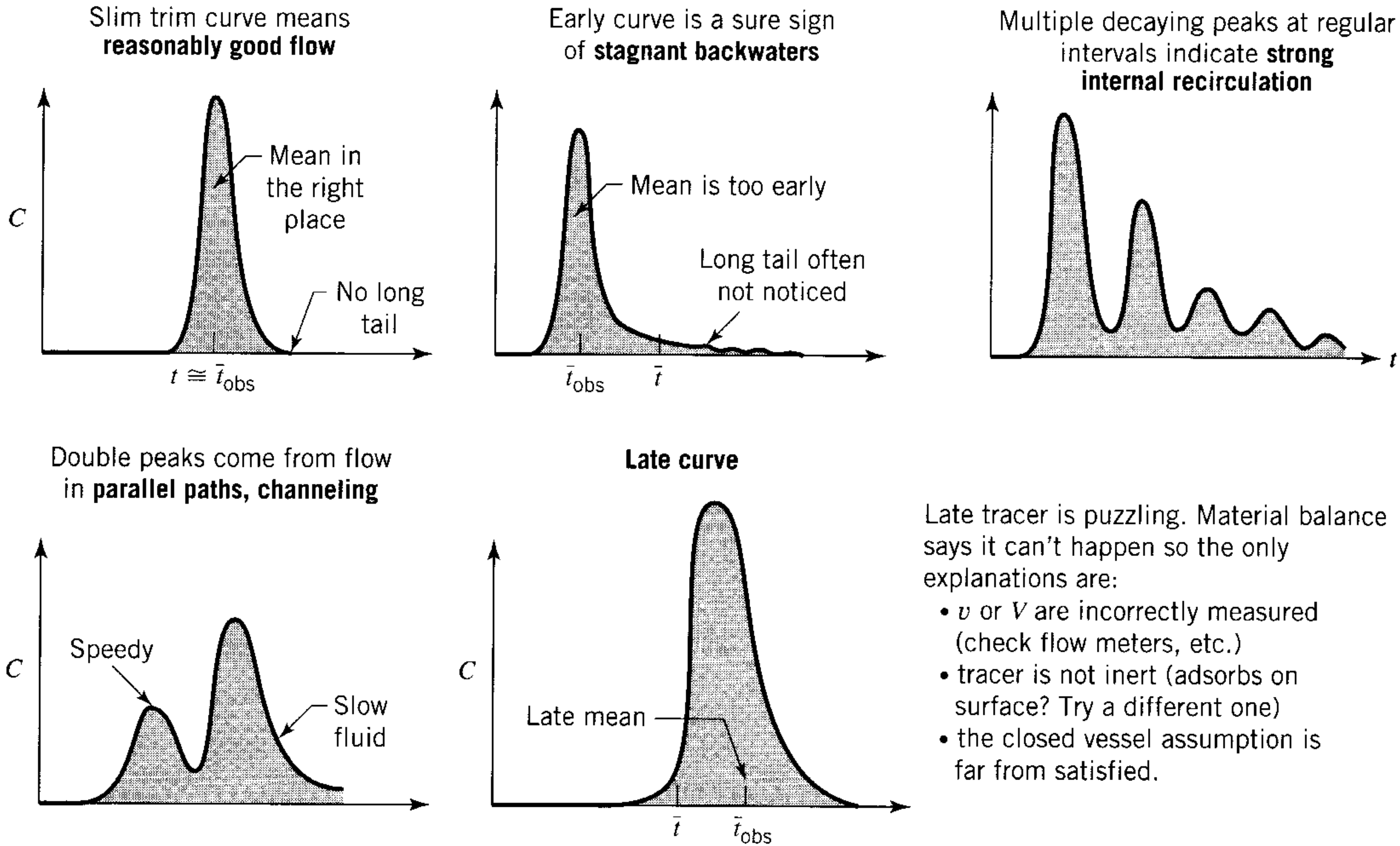


Figure 12.3 Misbehaving plug flow reactors.

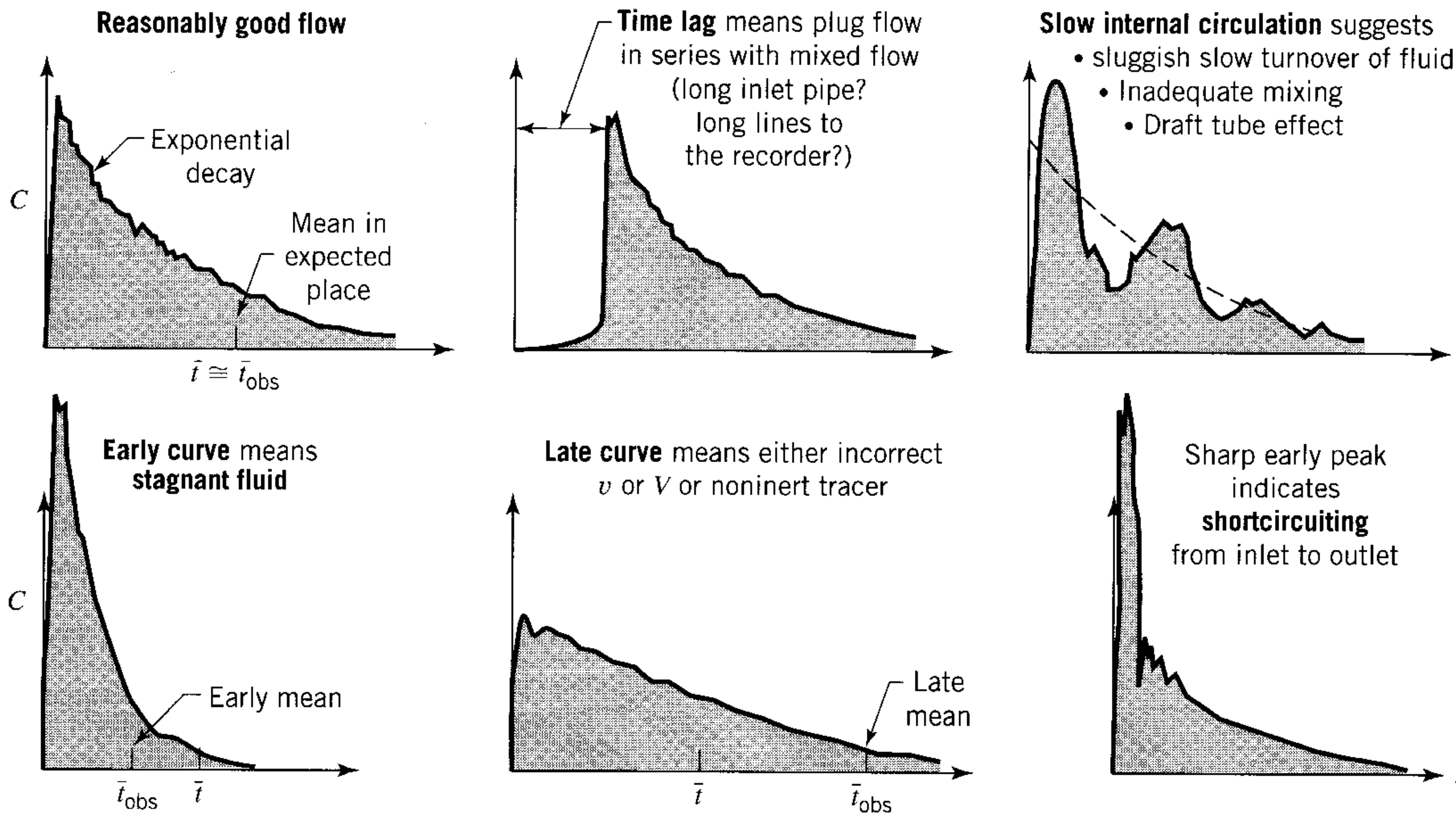


Figure 12.4 Misbehaving mixed flow reactors.

EXAMPLE 12.1 BEHAVIOR OF A G/L CONTACTOR

From the measured pulse tracer response curves (see figure), find the fraction of gas, of flowing liquid, and of stagnant liquid in the gas-liquid contactor shown in Fig. E12.1.

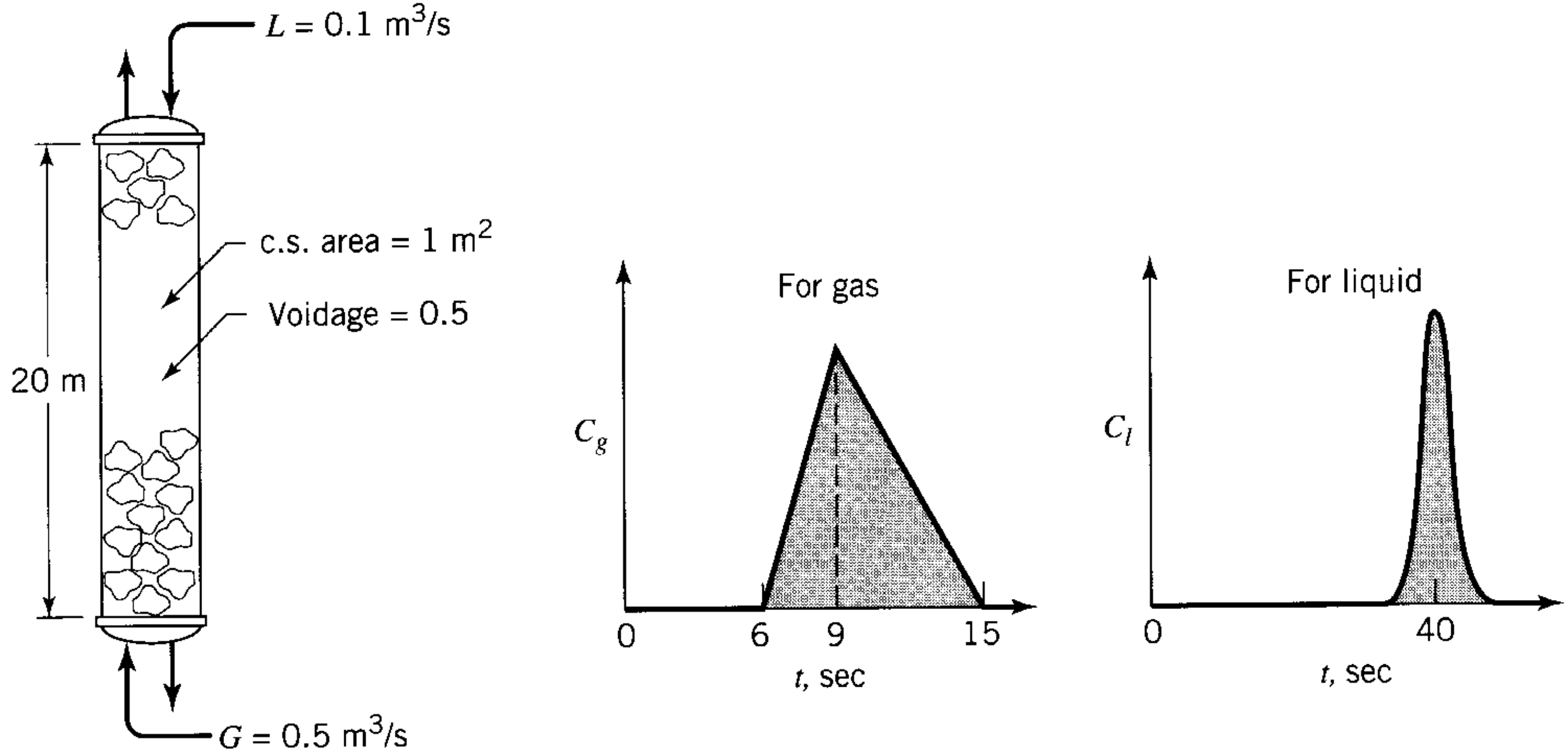


Figure E12.1

SOLUTION

To find V_g, V_l, and V_{stag}, first calculate $\bar{t}_g$ and $\bar{t}_l$ from the tracer curves. Thus from Fig. E12.1

$$\bar{t}_g = \frac{\sum tC}{\sum C} = \frac{8\,(9-6)(\mathrm{h}/2) + 11(15-9)(\mathrm{h}/2)}{(15-6)(\mathrm{h}/2)} = 10\ \mathrm{s}$$

and

$$\bar{t}_l = 40\ \mathrm{s}.$$

Therefore

$$V_g = \bar{t}_g v_g = (10)(0.5) = 5\mathrm{m}^3$$

$$V_l = \bar{t}_l v_l = 40(0.1) = 4\mathrm{m}^3$$

In terms of void volume

$$\left.\begin{aligned} \%\,\mathrm{G} &= 50\% \\ \%\,\mathrm{L} &= 40\% \\ \%\ \text{stagnant} &= 10\% \end{aligned}\right\} \leftarrow$$

EXAMPLE 12.2 CURING A MISBEHAVING REACTOR

At present our 6-m^3 tank reactor gives 75% conversion for the first order reaction $A \rightarrow R$. However, since the reactor is stirred with an underpowered paddle turbine, we suspect incomplete mixing and poor flow patterns in the vessel. A pulse tracer shows that this is so and gives the flow model sketched in Fig. E12.2. What conversion can we expect if we replace the stirrer with one powerful enough to ensure mixed flow?

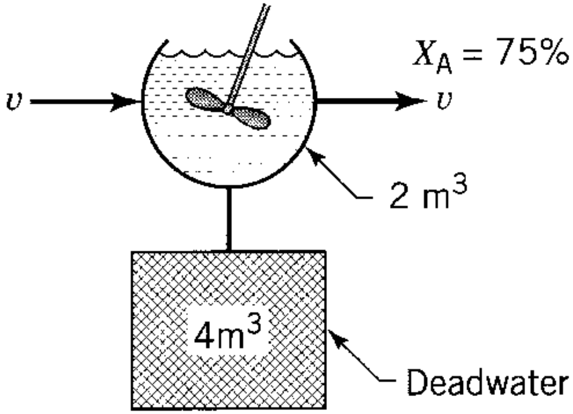


Figure E12.2

SOLUTION

Let subscript 1 represent today's reactor and subscript 2 represent the cured reactor. At present, from Chapter 5 for the MFR, we have

$$k\tau_1 = \frac{C_{A0} - C_A}{C_A} = \frac{C_{A0}}{C_A} - 1 = \frac{1}{0.25} - 1 = 3$$

But $k\tau_2 = 3\ k\tau_1 = 3 \times 3 = 9$

Therefore

$$\frac{C_{A2}}{C_{A0}} = \frac{1}{k\tau_2 + 1} = \frac{1}{9 + 1} = 0.1$$

or

$$\underline{\underline{X_{A2} = 90\%}}$$