Chapter 15 The Convection Model for Laminar Flow

- For a viscous fluid, one has laminar flow with its characteristics parabolic velocity profile.
- The pure convection model assumes that each element of fluid slides past its neighbor with no interaction by molecular diffusion. Thus the spread in residence times is caused only by velocity variations

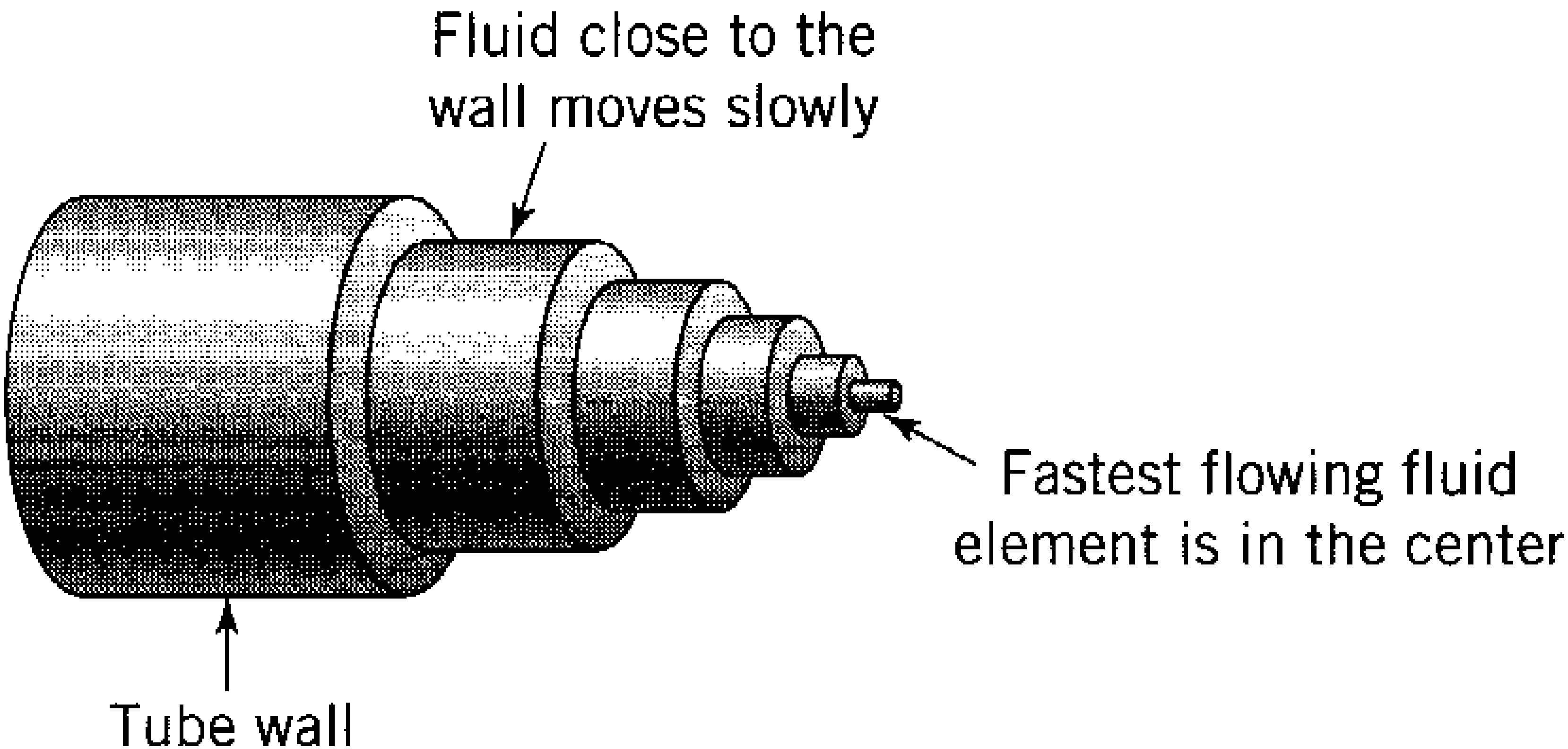

Figure 15.1 Flow of fluid according to the convection model.

- 15.1 The convection Model and Its RTD
- How tell from theory which model to use
- Depend on the fluid being used(Sc), the flow condition(Re), and vessel geometry.
- Note: for laminar flow only
- $\mathscr{D}$ is not D

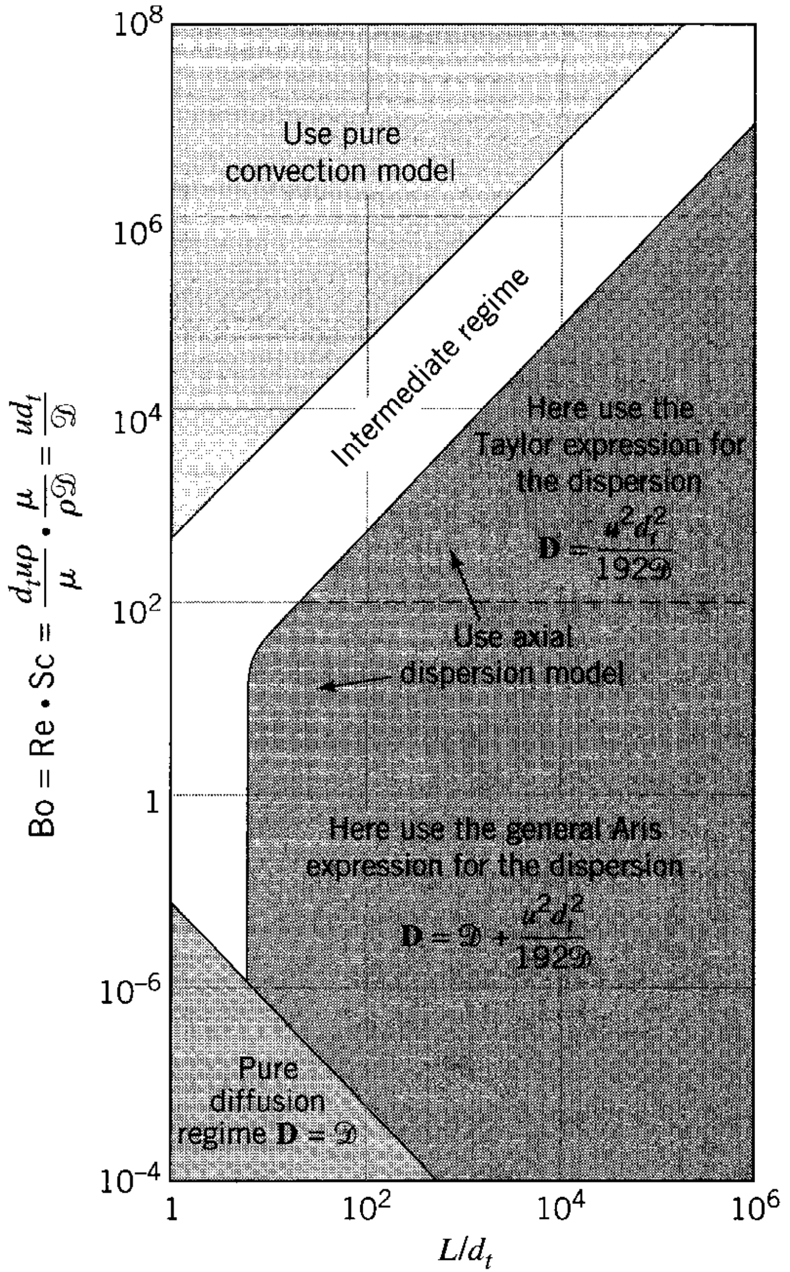

Figure 15.2 Map showing which flow models should be used in any situation.

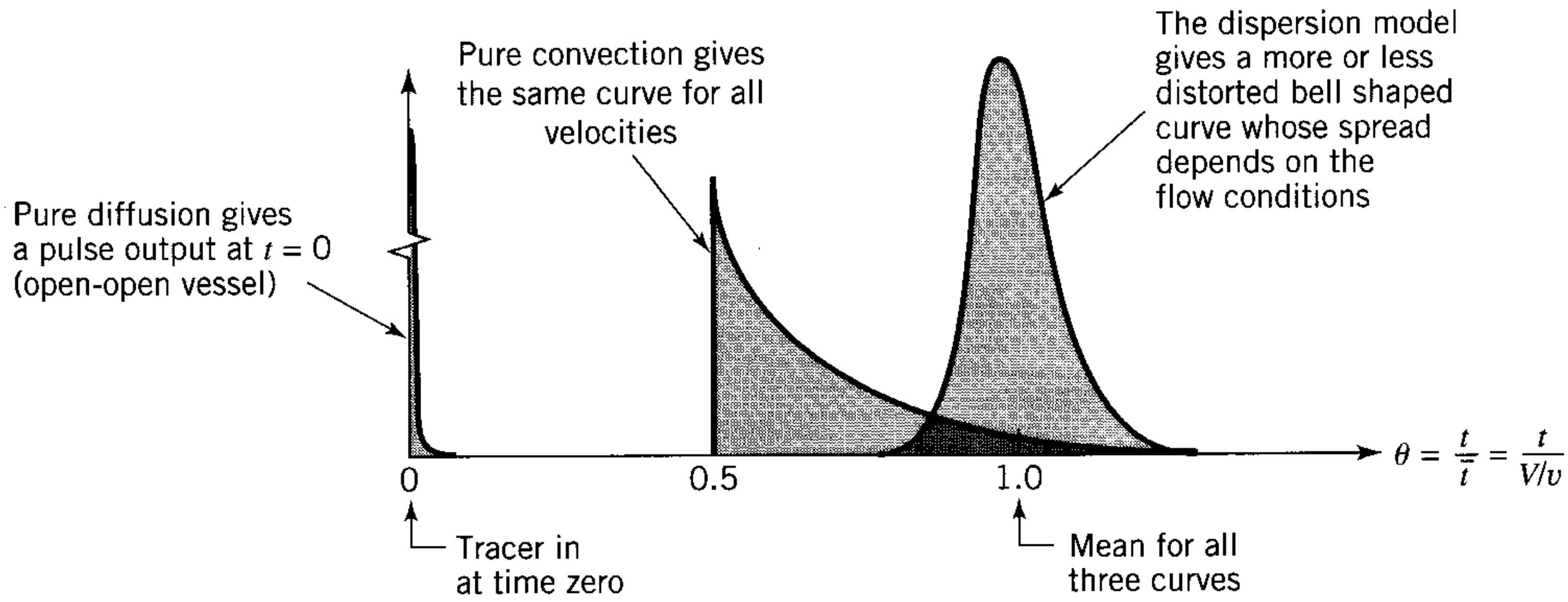

Figure 15.3 Comparison of the RTD of the three models.

It is very important to use the correct type of model because the RTD curves are completely different for different regimes.

How to tell from experiment which model to use

For dispersion model and tank-in-series model, $\sigma^2 \propto L$

For convection model, $\sigma \propto L$

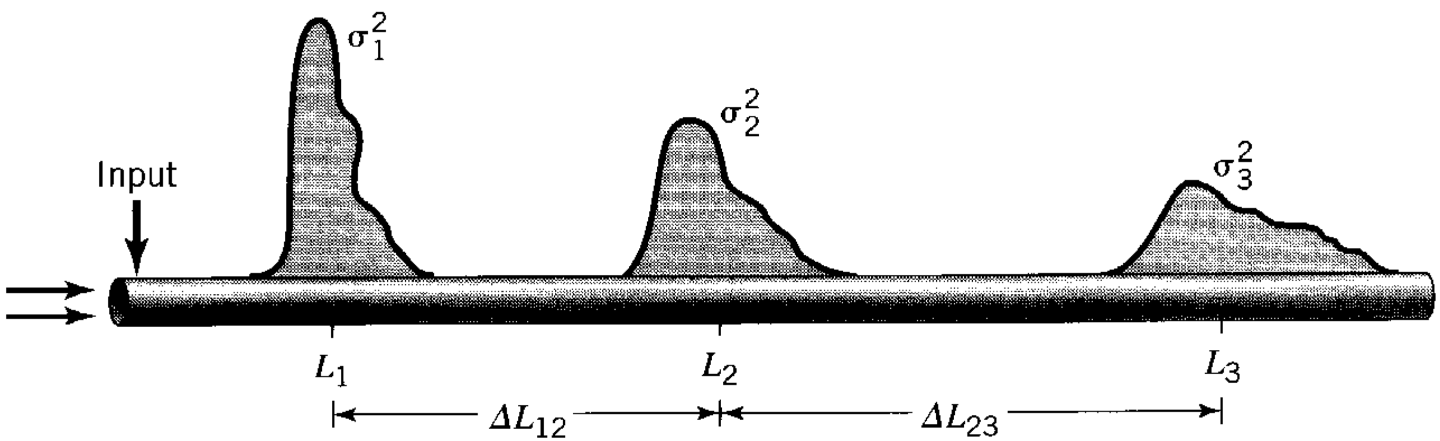

Figure 15.4 The changing spread of a tracer curve tells which model is the right one to use.

- Pulse response experiment and the E curve for laminar flow in pipes
- The shape of the response curve is strongly influenced by the way tracer is introduced into the flowing fluid, and how it is measured.
- You may inject or measure the tracer in two main ways, therefore we have four combinations of boundary conditions.

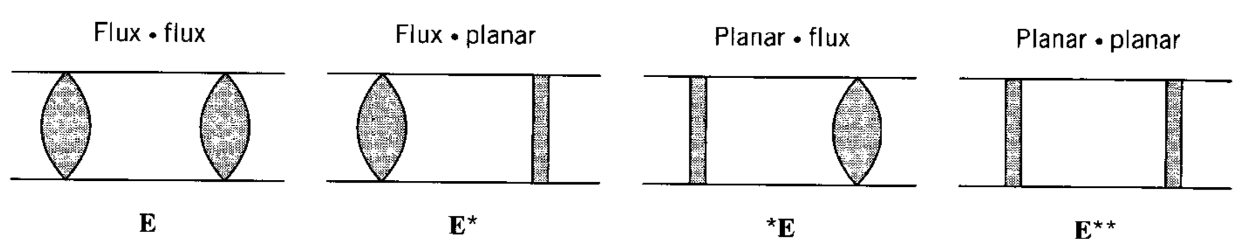

Figure 15.6 Various combinations of input-output methods.

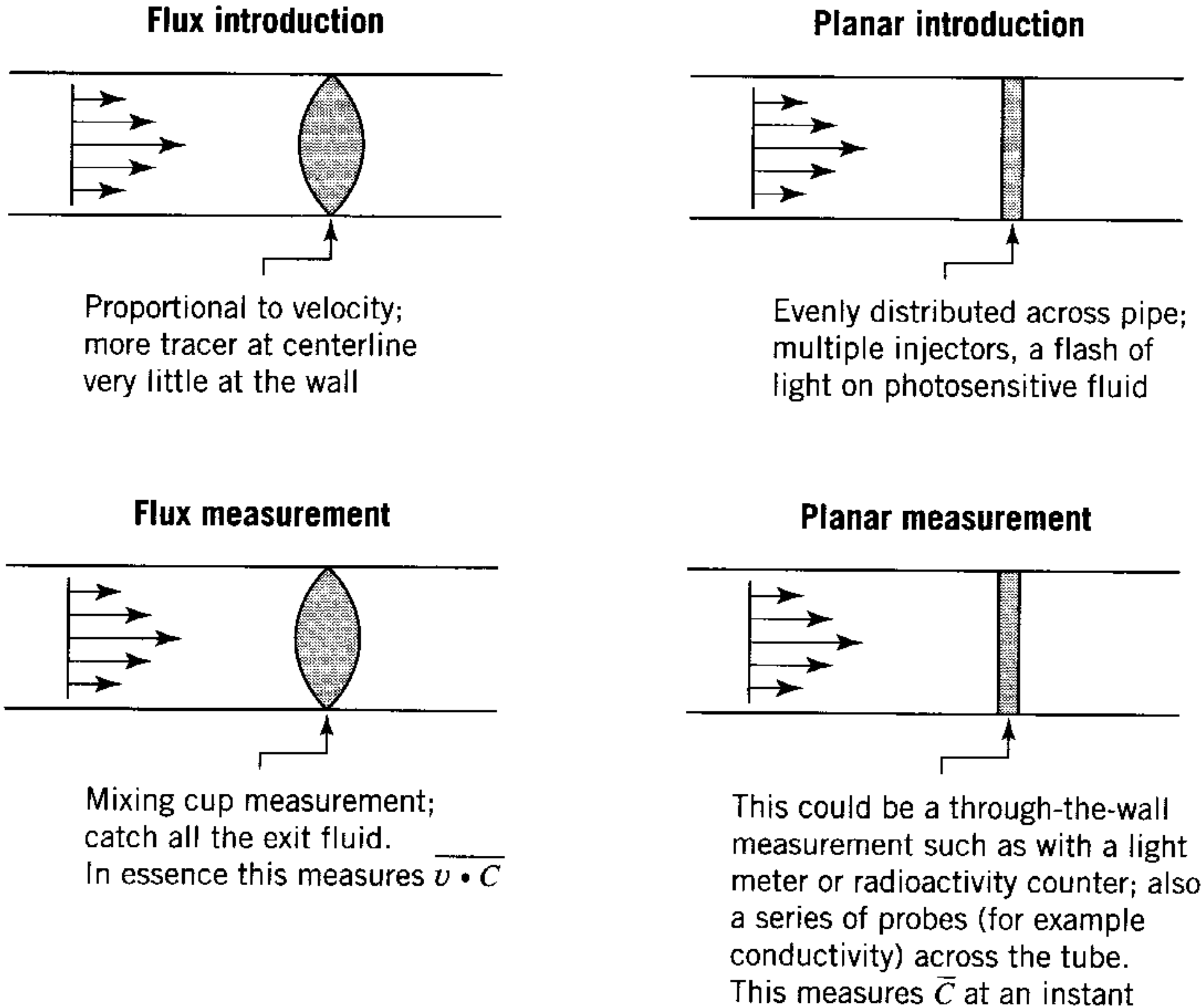

Figure 15.5 Various ways of introducing and measuring tracer.

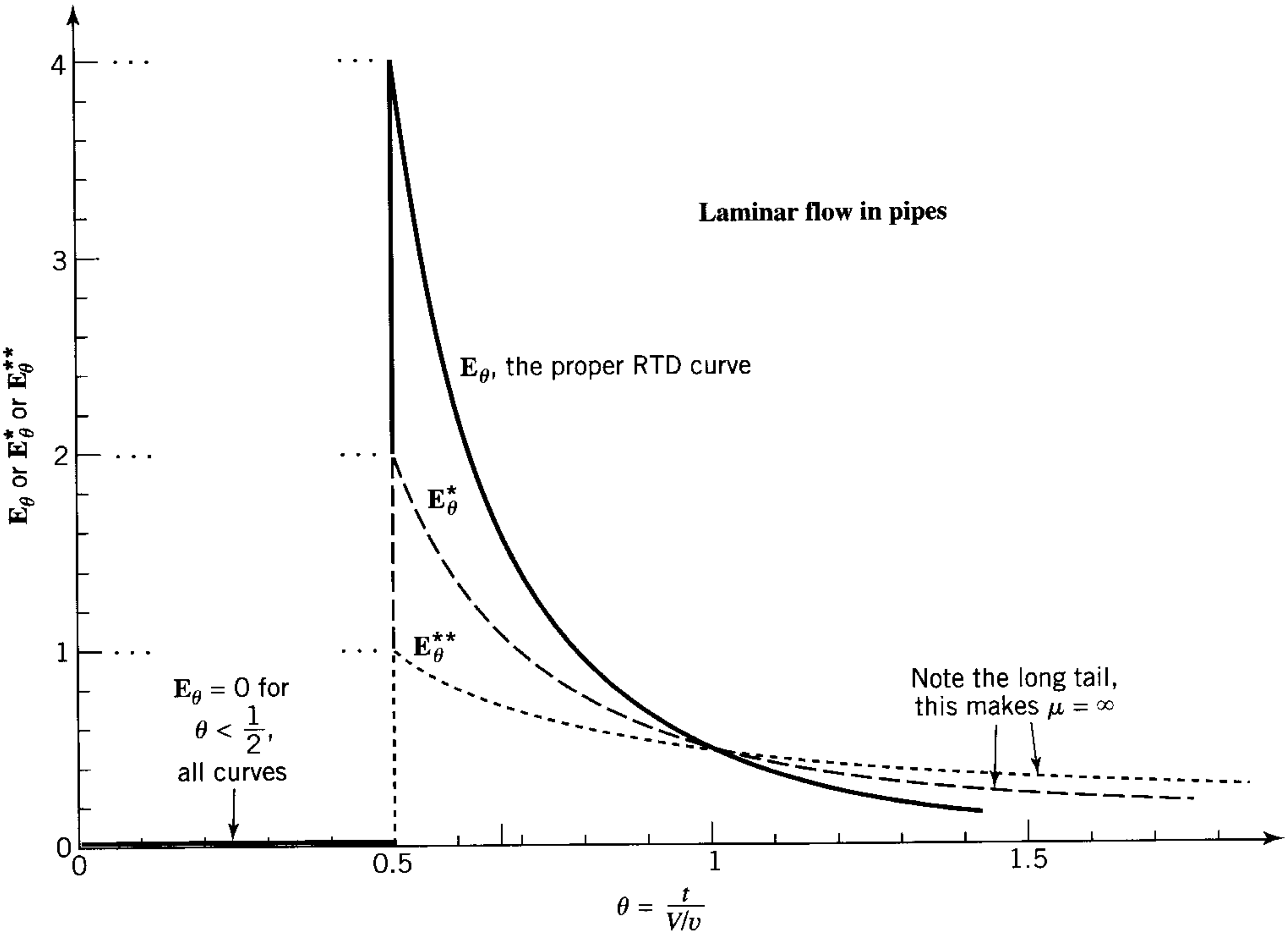

Figure 15.7 Note how different are the output curves depending on how you introduce and measure tracer.

$$\left.\begin{aligned} \mathbf{E} &= \frac{\bar{t}^2}{2t^3} \quad \text{for} \quad t \geq \frac{\bar{t}}{2} \\ \mathbf{E}_\theta &= \frac{1}{2\theta^3} \quad \text{for} \quad \theta \geq \frac{1}{2} \end{aligned} \quad \text{and} \quad \begin{aligned} \mu_t &= \bar{t} = \frac{V}{v} \\ \mu_\theta &= 1 \end{aligned} \right\}$$

$$\left.\begin{aligned} \mathbf{E}^* &= \frac{\bar{t}}{2t^2} \quad \text{for} \quad t \geq \frac{\bar{t}}{2} \\ \mathbf{E}^*_\theta &= \frac{1}{2\theta^2} \quad \text{for} \quad \theta \geq \frac{1}{2} \end{aligned} \quad \text{and} \quad \begin{aligned} \mu^* &= \infty \\ \bar{t} &= \frac{V}{v} \\ \theta &= t \Big/ \frac{V}{v} \end{aligned} \right\}$$

$$\left.\begin{aligned} \mathbf{E}^{**} &= \frac{1}{2t} \quad \text{for} \quad t \geq \frac{\bar{t}}{2} \\ \mathbf{E}^{**}_\theta &= \frac{1}{2\theta} \quad \text{for} \quad \theta \geq \frac{1}{2} \end{aligned} \quad \text{and} \quad \begin{aligned} \mu^{**} &= \infty \\ \bar{t} &= \frac{V}{v} \\ \theta &= t \Big/ \frac{V}{v} \end{aligned} \right\}$$

$$U = U_{\max}\left(1-\left(\frac{r}{R}\right)^2\right) = 2U_{avg}\left(1-\left(\frac{r}{R}\right)^2\right) = \frac{2}{\pi}\frac{v_0}{R^2}\left(1-\left(\frac{r}{R}\right)^2\right)$$

$$t(r) = \frac{L}{U(r)} = \frac{\pi R^2 L}{v_0}\frac{1}{2\left(1-\left(\frac{r}{R}\right)^2\right)} = \frac{\bar{t}}{2\left(1-\left(\frac{r}{R}\right)^2\right)}$$

$$\frac{dU}{v_0} = \frac{U(r)2\pi r dr}{v_0}$$

$U-[m/s]$ $\quad$ $v_0-[m^3/s]$ $\quad$ $t[s]$ $\quad$ $R-[m]$

$$dt = \frac{\bar{t}}{2R^2}\frac{2rdr}{\left(1-\left(\frac{r}{R}\right)^2\right)^2} = \frac{4}{\bar{t}R^2}\left(\frac{\frac{\bar{t}}{2}}{1-\left(\frac{r}{R}\right)^2}\right)^2 rdr \quad dt = \frac{4t^2}{\bar{t}R^2}rdr$$

The fraction of fluid spending between time t and t + dt in the reactor

$$\frac{dU}{v_0} = \frac{L}{t}\left(\frac{2\pi rdr}{v_0}\right) = \frac{L}{t}\left(\frac{2\pi}{v_0}\right)\frac{\bar{t}R^2}{4t^2}dt = \underline{\frac{\bar{t}^2}{2t^3}}dt$$

The minimium time for the fluid may spend in the reactor is :

$$t = \frac{L}{U_{\max}} = \frac{L\left(\pi R^2\right)}{2U_{avg}\left(\pi R^2\right)} = \frac{V_r}{2v_0} = \underline{\frac{\bar{t}}{2}}$$

Step response experiments and the F curve for laminar flow in pipes

The input always represents the flux input, while the output can be either planar or flux.

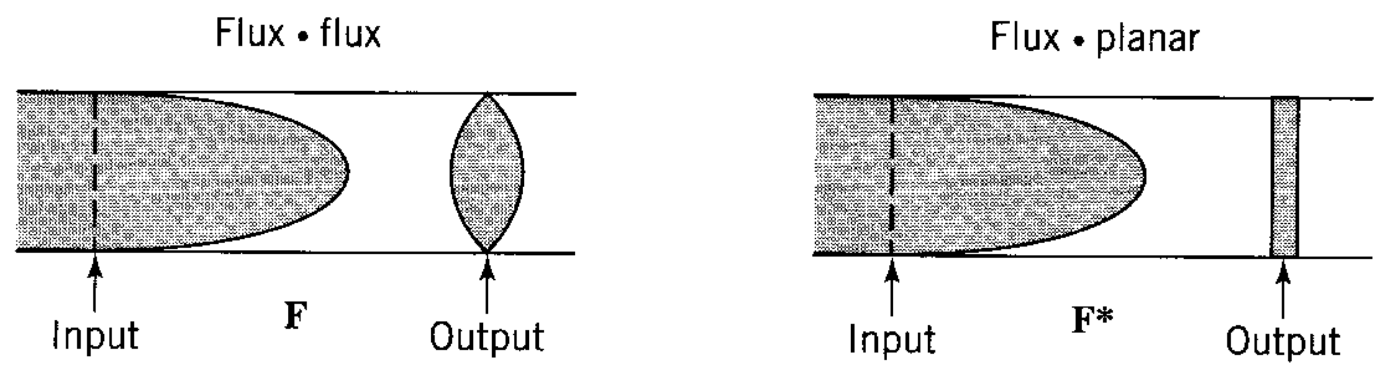

Figure 15.8 Two different ways of measuring the output curves.

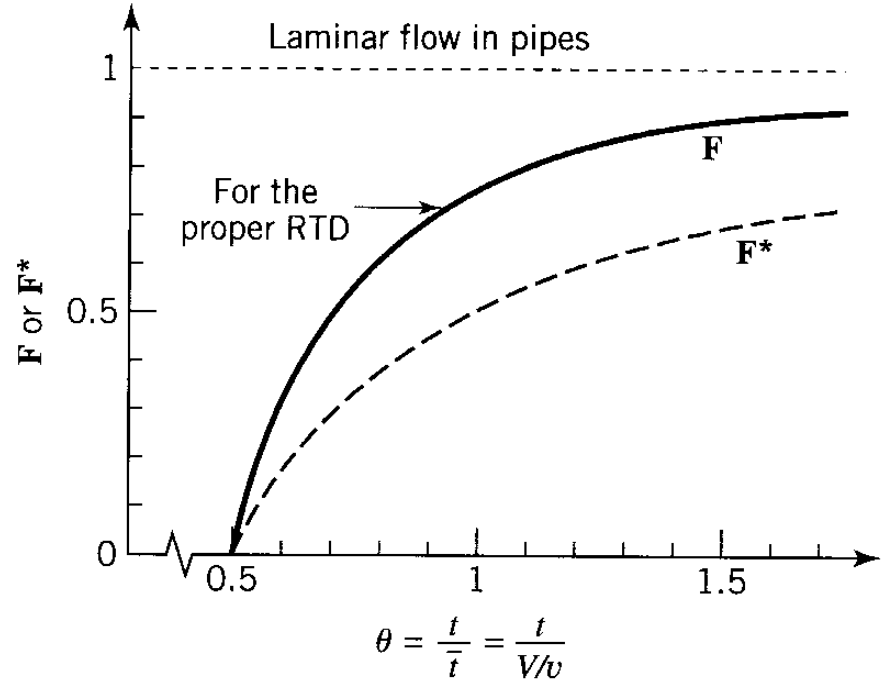

Figure 15.9 Different ways of measuring the output gives different **F** curves.

$$\left.\begin{aligned} F &= 1-\frac{1}{4\theta^2} \quad \text{for} \quad \theta \ge \frac{1}{2} \\ F^* &= 1-\frac{1}{2\theta} \quad \text{for} \quad \theta \ge \frac{1}{2} \end{aligned}\right\} \quad \text{where } \theta = \frac{t}{\left(\frac{V}{v}\right)}$$

The relationshap between F and F* is similar between E and F

15.2 Chemical Conversion in Laminar Flow Reactor

- In the pure convection regime each element of fluid follows its own streamline with no intermixing with neighboring elements.
- In essence this gives macrofluid behavior.
- Because a laminar flow has a determined RTD curve, the conversion of a reaction is depended only on the order for reaction and average residence time.

$$\frac{C_A}{C_{A0}} = \int_0^\infty \left(\frac{C_A}{C_{A0}} \right)_{\substack{\text{element}\\ \text{of fluid}}} Edt$$

$$\frac{C_A}{C_{A0}} = 1 - \frac{kt}{C_{A0}} \quad \text{for t} \le \frac{C_{A0}}{k} \qquad \text{for zero order reaction}$$

$$\frac{C_A}{C_{A0}} = e^{-kt} \qquad \text{for first order reaction}$$

$$\frac{C_A}{C_{A0}} = \frac{1}{1+kC_{A0}t} \qquad \text{for second order reaction}$$

for zero order reaction $$\frac{C_A}{C_{A0}} = \left(1 - \frac{k\bar{t}}{2C_{A0}}\right)$$

for first order reaction

$$\frac{C_A}{C_{A0}} = \frac{\bar{t}^2}{2}\int_{\frac{\bar{t}}{2}}^{\infty}\frac{e^{-kt}}{t^3}dt = y^2 ei(y) + (1-y)e^{-y}$$

$$y = \frac{k\bar{t}}{2} \qquad \left(ei(y) = \int_y^{\infty}\frac{e^x}{x}dx\right)$$

for second order reaction

$$\frac{C_A}{C_{A0}} = 1 - kC_{A0}\bar{t}\left(1 - \frac{kC_{A0}\bar{t}}{2}\ln\left(1 + \frac{2}{kC_{A0}\bar{t}}\right)\right)$$

The relationship between laminar flow and plug flow

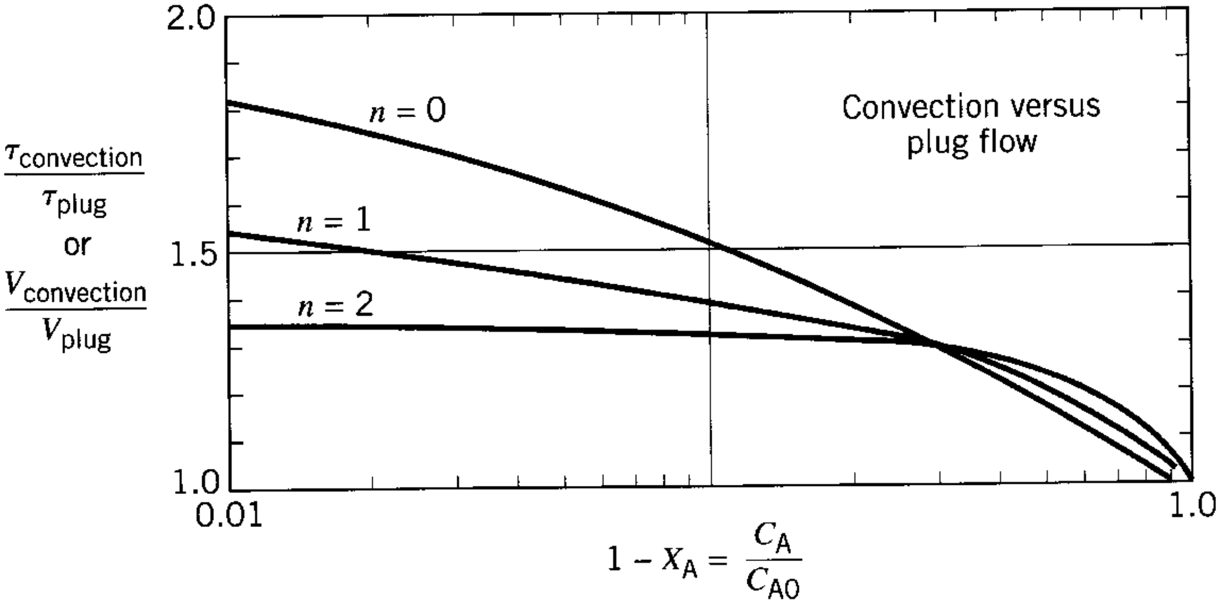

Figure 15.10 Convective flow lowers conversion compared to plug flow.

This graph shows that even at high X_A convective flow does not drastically lower reactor performance.

It is differ from the dispersion model and tank-in-series model.

- Multiple reaction in laminar flow
- Consider a two-step first order irreversible reaction in series

$$A \xrightarrow{k_1} R \xrightarrow{k_2} S$$

- Because laminar flow represents a deviation from plug flow, the amount of intermediate formed will be somewhat less than for plug flow.

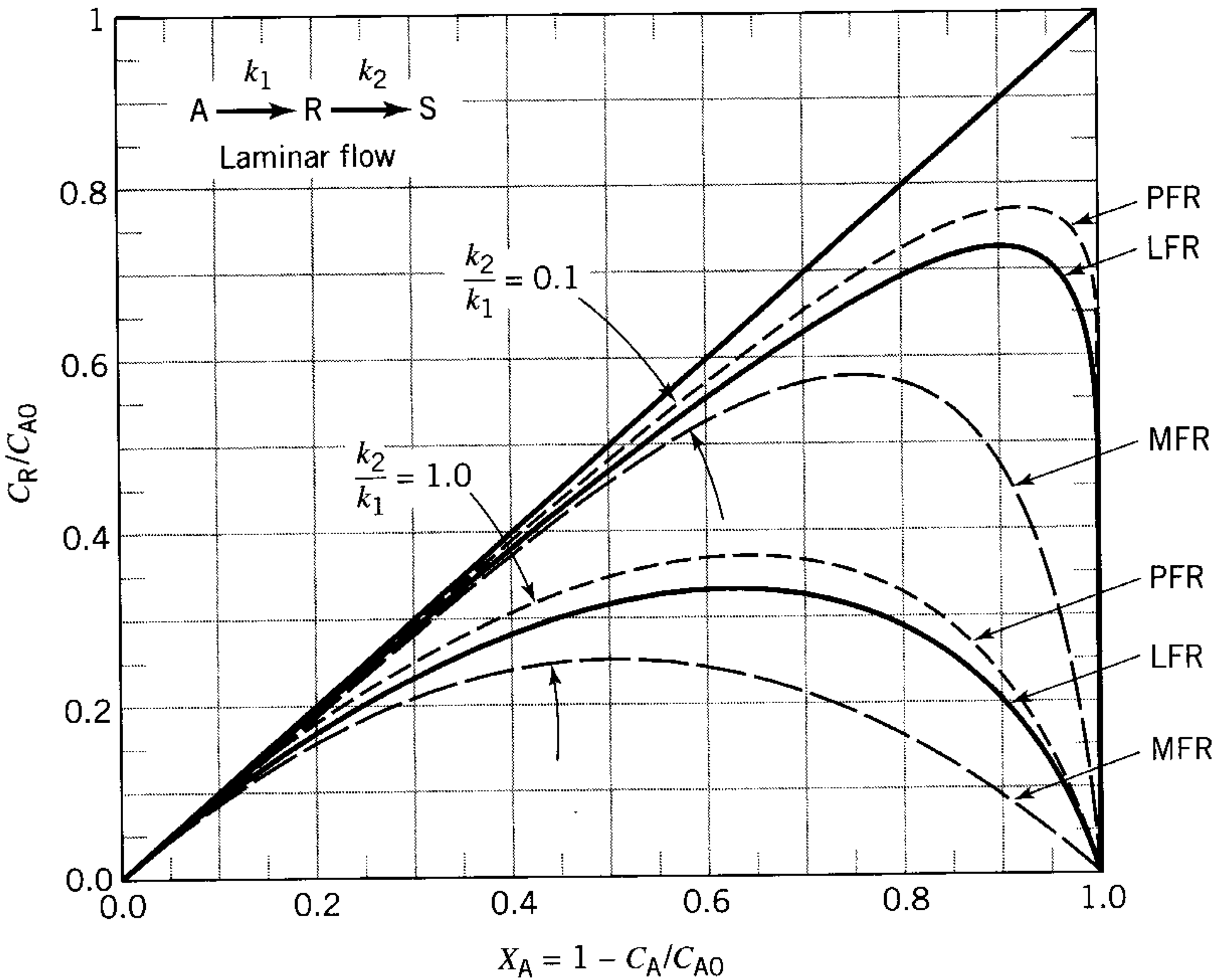

Figure 15.11 Typical product distribution curves for laminar flow compared with the curves for plug flow (Fig. 8.13) and mixed flow (Fig. 8.14).