# Differential and integral forms of the general transport equations

Continuity 
$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0$$
 (2.4)
$$x\text{-momentum} \qquad \frac{\partial(\rho u)}{\partial t} + \operatorname{div}(\rho u \mathbf{u}) = -\frac{\partial p}{\partial x} + \operatorname{div}(\mu \operatorname{grad} u) + S_{Mx}$$
 (2.37a)
$$y\text{-momentum} \qquad \frac{\partial(\rho v)}{\partial t} + \operatorname{div}(\rho v \mathbf{u}) = -\frac{\partial p}{\partial y} + \operatorname{div}(\mu \operatorname{grad} v) + S_{My}$$
 (2.37b)
$$z\text{-momentum} \qquad \frac{\partial(\rho w)}{\partial t} + \operatorname{div}(\rho w \mathbf{u}) = -\frac{\partial p}{\partial z} + \operatorname{div}(\mu \operatorname{grad} w) + S_{Mz}$$
 (2.37c)
$$Energy \qquad \frac{\partial(\rho i)}{\partial t} + \operatorname{div}(\rho i \mathbf{u}) = -p \operatorname{div} \mathbf{u} + \operatorname{div}(k \operatorname{grad} T) + \Phi + S_i$$
 (2.38)
$$Equations \qquad p = p(\rho, T) \text{ and } i = i(\rho, T)$$
 (2.28)

e.g. perfect gas  $p = \rho RT$  and  $i = C_v T$ 

(2.29)

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 (2.4)   
  $x$ -momentum 
$$\frac{\partial(\rho u)}{\partial t} + \operatorname{div}(\rho u \mathbf{u}) = -\frac{\partial p}{\partial x} + \operatorname{div}(\mu \operatorname{grad} u) + S_{Mx}$$
 (2.37a)   
  $y$ -momentum 
$$\frac{\partial(\rho v)}{\partial t} + \operatorname{div}(\rho v \mathbf{u}) = -\frac{\partial p}{\partial y} + \operatorname{div}(\mu \operatorname{grad} v) + S_{My}$$
 (2.37b)   
  $z$ -momentum 
$$\frac{\partial(\rho w)}{\partial t} + \operatorname{div}(\rho w \mathbf{u}) = -\frac{\partial p}{\partial z} + \operatorname{div}(\mu \operatorname{grad} w) + S_{Mz}$$
 (2.37c)   
 Energy 
$$\frac{\partial(\rho i)}{\partial t} + \operatorname{div}(\rho i \mathbf{u}) = -p \operatorname{div} \mathbf{u} + \operatorname{div}(k \operatorname{grad} T) + \Phi + S_i$$
 (2.38)

$$\frac{\partial(\rho\phi)}{\partial t} + \operatorname{div}(\rho\phi\mathbf{u}) = \operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi}$$

$$\frac{\partial(\rho\phi)}{\partial t} + \operatorname{div}(\rho\phi\mathbf{u}) = \operatorname{div}(\Gamma \text{ grad } \phi) + S_{\phi}$$

Rate of increase Net rate of flow Rate of increase Rate of increase of  $\phi$  of fluid + of  $\phi$  out of = of  $\phi$  due to + of  $\phi$  due to element fluid element diffusion sources

$$\frac{\partial(\rho\phi)}{\partial t} + \operatorname{div}(\rho\phi\mathbf{u}) = \operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi}$$

$$\int \frac{\partial(\rho\phi)}{\partial t} dV + \int \operatorname{div}(\rho\phi\mathbf{u}) dV = \int \operatorname{div}(\Gamma \operatorname{grad} \phi) dV + \int S_{\phi} dV$$

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$$\frac{\partial(\rho\phi)}{\partial t} + \operatorname{div}(\rho\phi\mathbf{u}) = \operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi}$$

$$\int_{\text{cv}} \frac{\partial(\rho\phi)}{\partial t} dV + \int_{\text{cv}} \operatorname{div}(\rho\phi\mathbf{u}) dV = \int_{\text{cv}} \operatorname{div}(\Gamma \operatorname{grad} \phi) dV + \int_{\text{cv}} S_{\phi} dV$$

$$\int_{CV} \operatorname{div}(\mathbf{a}) dV = \int_{A} \mathbf{n} \cdot \mathbf{a} dA$$

$$\frac{\partial(\rho\phi)}{\partial t} + \operatorname{div}(\rho\phi\mathbf{u}) = \operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi}$$

$$\int_{\operatorname{cv}} \frac{\partial(\rho\phi)}{\partial t} dV + \int_{\operatorname{cv}} \operatorname{div}(\rho\phi\mathbf{u}) dV = \int_{\operatorname{cv}} \operatorname{div}(\Gamma \operatorname{grad} \phi) dV + \int_{\operatorname{cv}} S_{\phi} dV$$

$$\frac{\partial}{\partial t} \left(\int_{\operatorname{CV}} \rho\phi dV\right) + \int_{\operatorname{d}} \mathbf{n} \cdot (\rho\phi\mathbf{u}) dA = \int_{\operatorname{d}} \mathbf{n} \cdot (\Gamma \operatorname{grad} \phi) dA + \int_{\operatorname{cv}} S_{\phi} dV$$

$$\int_{CV} \operatorname{div}(\mathbf{a}) dV = \int_{A} \mathbf{n} \cdot \mathbf{a} dA$$

$$\frac{\partial}{\partial t} \left( \int_{CV} \rho \phi dV \right) + \int_{A} \mathbf{n} \cdot (\rho \phi \mathbf{u}) dA = \int_{A} \mathbf{n} \cdot (\Gamma \operatorname{grad} \phi) dA + \int_{CV} S_{\phi} dV$$

Net rate of decrease Rate of increase of  $\phi$  due to of  $\phi$  inside the + convection across = due to diffusion control volume the control volume across the control boundaries

Net rate of increase of  $\phi$ volume boundaries

Net rate of  $_{+}$  creation of  $\phi$ inside the control volume

$$\frac{\partial(\rho\phi)}{\partial t} + \operatorname{div}(\rho\phi\mathbf{u}) = \operatorname{div}(\Gamma \text{ grad } \phi) + S_{\phi}$$

$$\frac{\partial(\rho\phi)}{\partial t} + \operatorname{div}(\rho\phi\mathbf{u}) = \operatorname{div}(\Gamma \text{ grad } \phi) + S_{\phi}$$

$$\frac{\partial(\rho\phi)}{\partial t} + \operatorname{div}(\rho\phi\mathbf{u}) = \operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi}$$

$$0 \qquad 0$$

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$$0 \qquad 0$$

$$\operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi} = 0$$

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The control volume integration, which forms the key step of the finite volume method that distinguishes it from all other CFD techniques, yields the following form:

$$\int_{CV} \operatorname{div}(\Gamma \operatorname{grad} \phi) dV + \int_{CV} S_{\phi} dV$$

$$= \int_{A} \mathbf{n} \cdot (\Gamma \operatorname{grad} \phi) dA + \int_{CV} S_{\phi} dV = 0$$
(4.2)

# $\operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi} = 0$

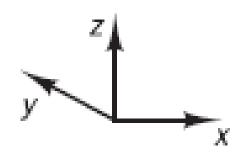
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$$\int_{CV} \operatorname{div}(\Gamma \operatorname{grad} \phi) dV + \int_{CV} S_{\phi} dV$$

$$= \int_{A} \mathbf{n} \cdot (\Gamma \operatorname{grad} \phi) dA + \int_{CV} S_{\phi} dV = 0 \tag{4.2}$$

Net rate of increase of  $\phi$  due to diffusion across the control volume boundaries

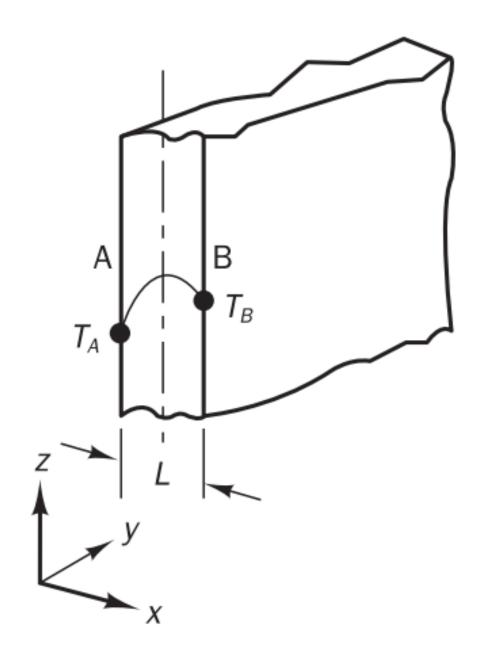
Net rate of creation of  $\phi$  inside the control volume

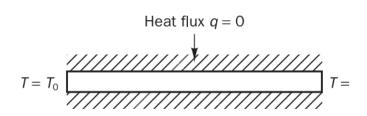


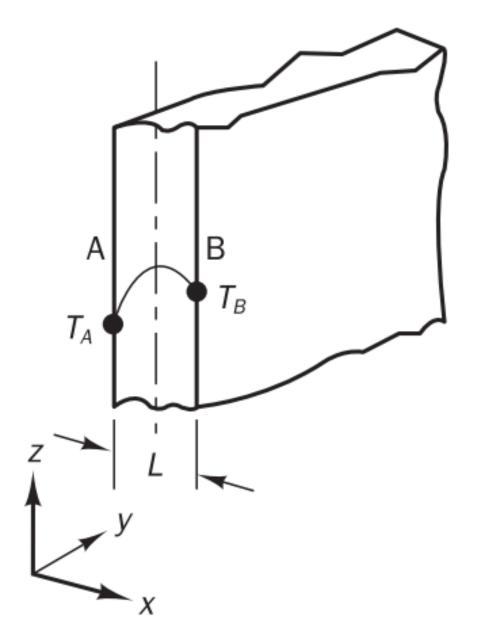
$$\operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi} = 0$$

$$\operatorname{div}(\Gamma \operatorname{grad} \phi) + S_{\phi} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \left( \Gamma \frac{\mathrm{d}\phi}{\mathrm{d}x} \right) + S = 0$$

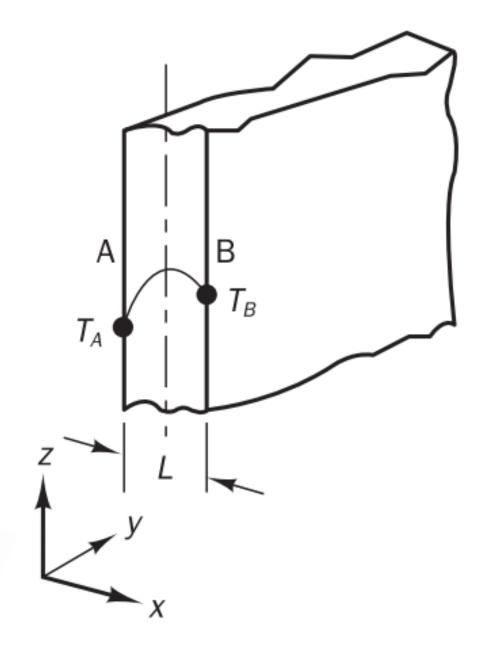




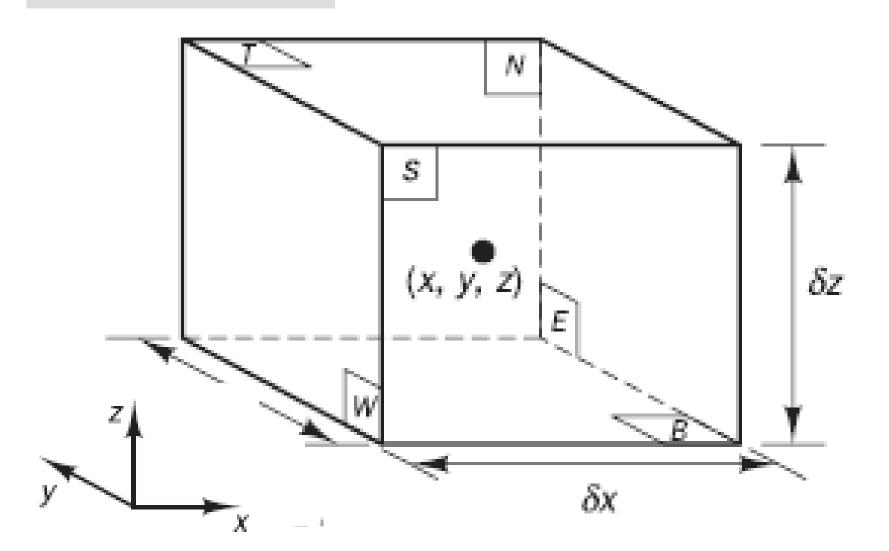


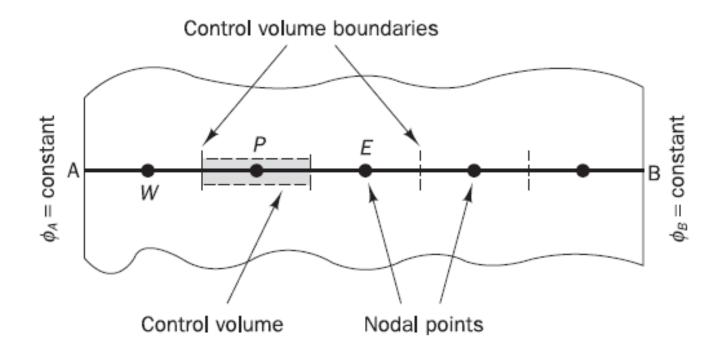
Finite volume method for onedimensional steady state diffusion

Heat flux q=0



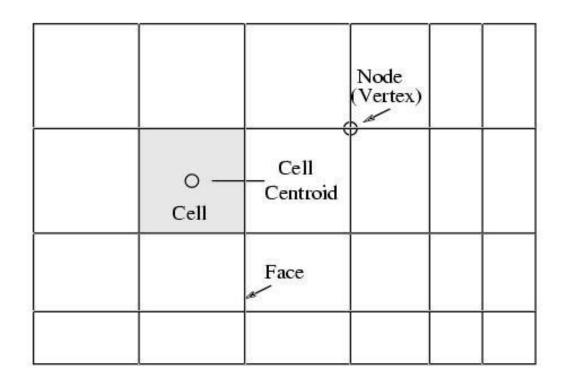
Finite volume method for onedimensional steady state diffusion





## Step 1: Grid generation

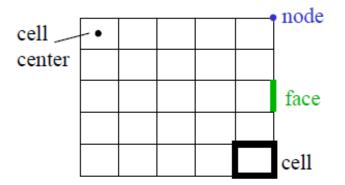
## Mesh Terminology



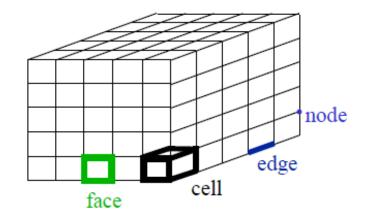
- Node-based finite volume scheme: φ stored at vertex
- Cell-based finite volume scheme: φ stored at cell centroid

## **Terminology**

- Cell = control volume into which domain is broken up.
- Node = grid point.
- Cell center = center of a cell.
- Edge = boundary of a face.
- Face = boundary of a cell.
- Zone = grouping of nodes, faces, and cells:
  - Wall boundary zone.
  - Fluid cell zone.
- Domain = group of node, face and cell zones.



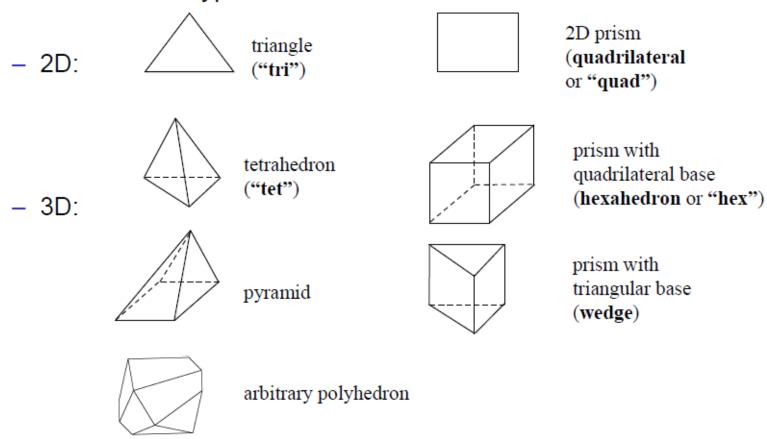
2D computational grid



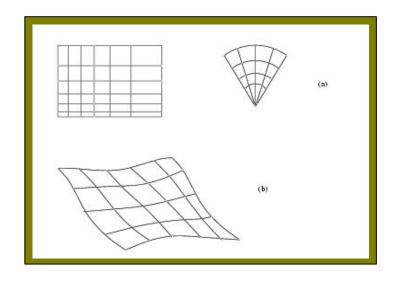
3D computational grid

#### Typical cell shapes

- Many different cell/element and grid types are available. Choice depends on the problem and the solver capabilities.
- Cell or element types:



# Mesh Types

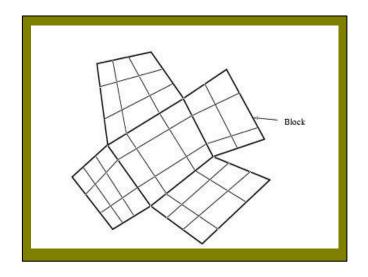




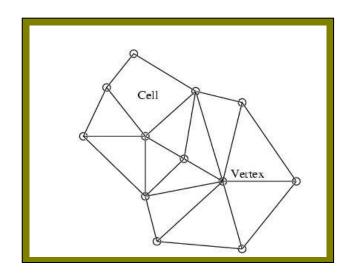
Regular and body-fitted meshes

Stair-stepped representation of complex geometry

## Mesh types (cont'd)



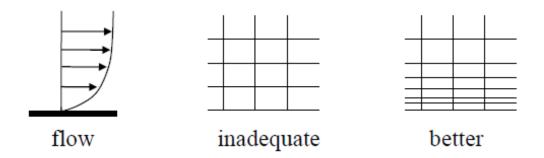
Blockstructured meshes



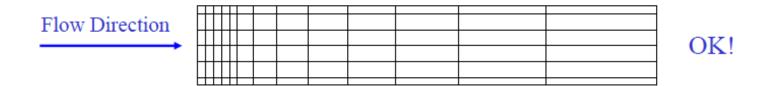
Unstructured meshes

## Grid design guidelines: resolution

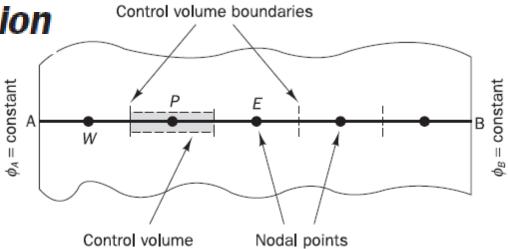
Pertinent flow features should be adequately resolved.

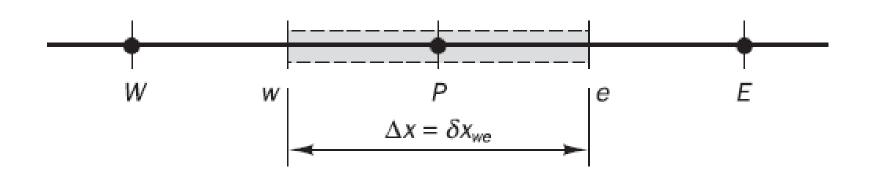


- Cell aspect ratio (width/height) should be near one where flow is multi-dimensional.
- Quad/hex cells can be stretched where flow is fully-developed and essentially one-dimensional.



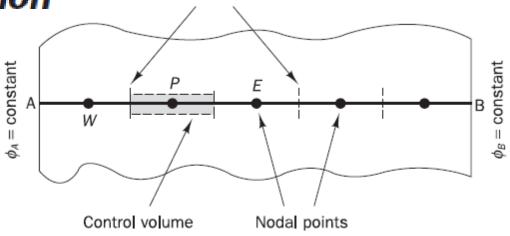
Step 1: Grid generation

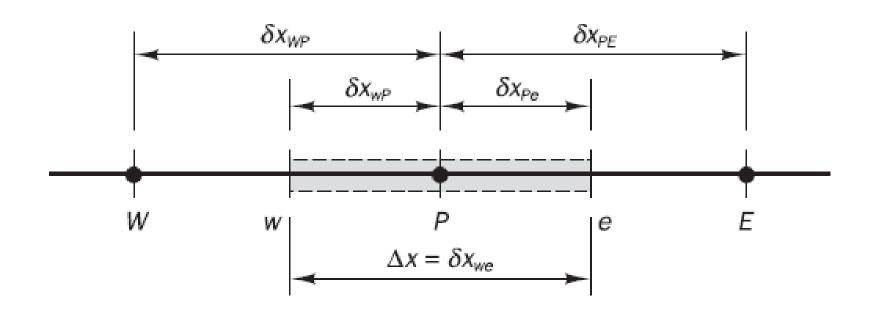




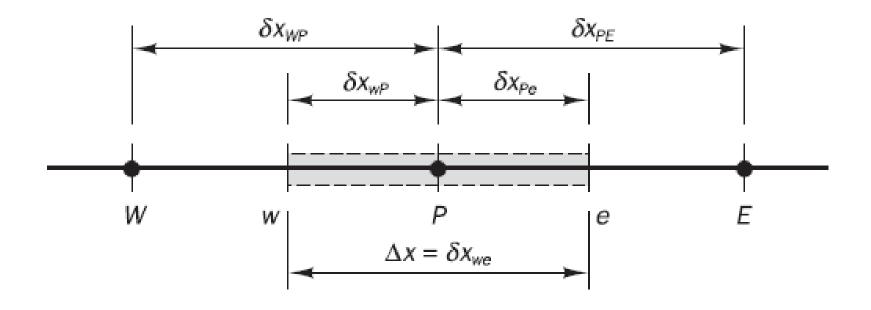
## Step 1: Grid generation

Control volume boundaries



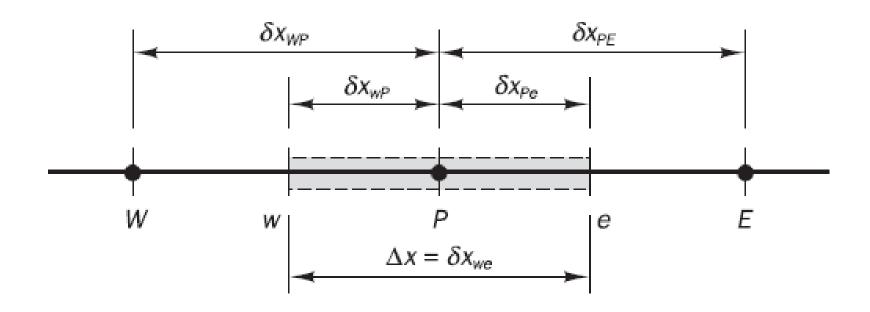


$$\frac{\mathrm{d}}{\mathrm{d}x} \left( \Gamma \frac{\mathrm{d}\phi}{\mathrm{d}x} \right) + S = 0$$



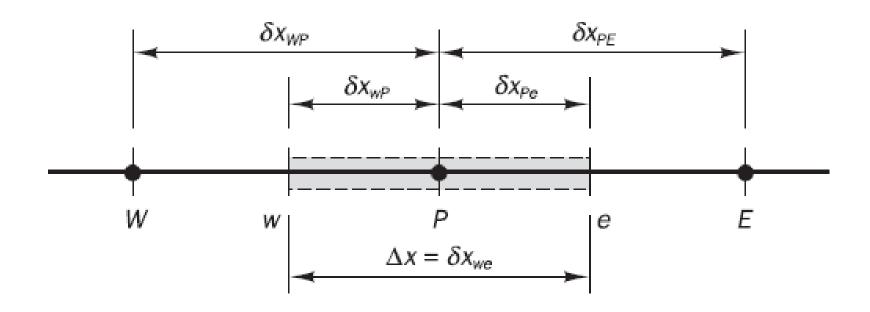
$$\int_{\Delta V} \frac{\mathrm{d}}{\mathrm{d}x} \left( \Gamma \frac{\mathrm{d}\phi}{\mathrm{d}x} \right) \mathrm{d}V + \int_{\Delta V} S \mathrm{d}V = \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{e} - \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{m} + \bar{S}\Delta V = 0 \tag{4.4}$$

Here A is the cross-sectional area of the control volume face,  $\Delta V$  is the volume and  $\bar{S}$  is the average value of source S over the control volume. It is



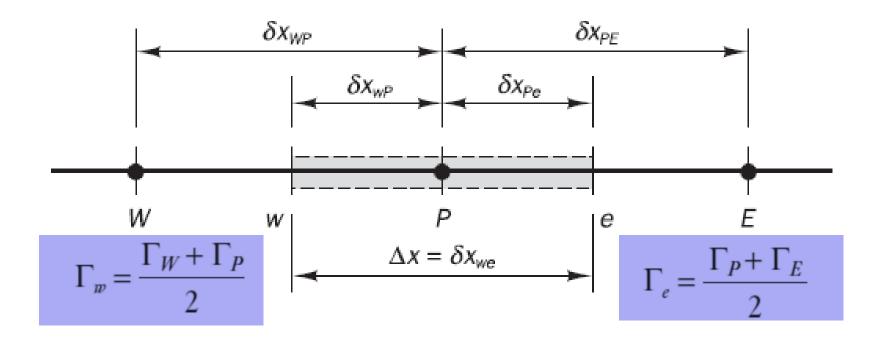
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$$\int_{\Delta V} \frac{\mathrm{d}}{\mathrm{d}x} \left( \Gamma \frac{\mathrm{d}\phi}{\mathrm{d}x} \right) \mathrm{d}V + \int_{\Delta V} S \mathrm{d}V = \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{e} - \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{w} + \bar{S}\Delta V = 0$$
 (4.4)

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$$\int_{\Delta V} \frac{\mathrm{d}}{\mathrm{d}x} \left( \Gamma \frac{\mathrm{d}\phi}{\mathrm{d}x} \right) \mathrm{d}V + \int_{\Delta V} S \mathrm{d}V = \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{e} - \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{p} + \bar{S}\Delta V = 0 \tag{4.4}$$

And the diffusive flux terms are evaluated as
$$\left(\Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x}\right)_{e} = \Gamma_{e} A_{e} \left(\frac{\phi_{E} - \phi_{P}}{\delta x_{PE}}\right)$$

$$\left(\Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x}\right)_{m} = \Gamma_{m} A_{m} \left(\frac{\phi_{P} - \phi_{W}}{\delta x_{WP}}\right)$$

$$W$$

$$W$$

$$P$$

$$\Delta x = \delta x_{We}$$

$$\Gamma_{e} = \frac{\Gamma_{W} + \Gamma_{P}}{2}$$

$$\Gamma_{e} = \frac{\Gamma_{P} + \Gamma_{E}}{2}$$

$$\int_{\Delta V} \frac{\mathrm{d}}{\mathrm{d}x} \left( \Gamma \frac{\mathrm{d}\phi}{\mathrm{d}x} \right) \mathrm{d}V + \int_{\Delta V} S \mathrm{d}V = \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{e} - \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{w} + \bar{S}\Delta V = 0 \qquad (4.4)$$

$$\bar{S}\Delta V = S_{u} + S_{p}\phi_{p}$$

And the diffusive flux terms are evaluated as 
$$\left(\Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x}\right)_{e} = \Gamma_{e} A_{e} \left(\frac{\phi_{E} - \phi_{P}}{\delta x_{PE}}\right)$$

$$\left(\Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x}\right)_{w} = \Gamma_{w} A_{w} \left(\frac{\phi_{P} - \phi_{W}}{\delta x_{WP}}\right)$$

$$W$$

$$P$$

$$\Delta x = \delta x_{we}$$

$$\Gamma_{e} = \frac{\Gamma_{W} + \Gamma_{P}}{2}$$

$$\Gamma_{e} = \frac{\Gamma_{P} + \Gamma_{E}}{2}$$

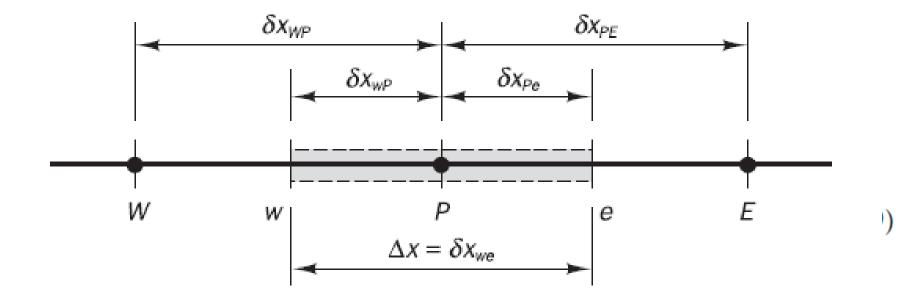
$$\int_{\Delta V} \frac{\mathrm{d}}{\mathrm{d}x} \left( \Gamma \frac{\mathrm{d}\phi}{\mathrm{d}x} \right) \mathrm{d}V + \int_{\Delta V} S \mathrm{d}V = \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{e} - \left( \Gamma A \frac{\mathrm{d}\phi}{\mathrm{d}x} \right)_{w} + \bar{S}\Delta V = 0 \tag{4.4}$$

Substitution of equations (4.6), (4.7) and (4.8) into equation (4.4) gives

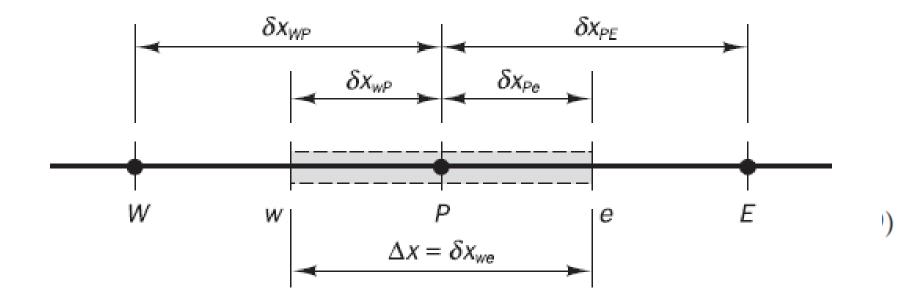
$$\Gamma_e A_e \left( \frac{\phi_E - \phi_P}{\delta x_{PE}} \right) - \Gamma_w A_w \left( \frac{\phi_P - \phi_W}{\delta x_{WP}} \right) + (S_u + S_p \phi_P) = 0 \tag{4.9}$$

This can be rearranged as

$$\left[ \left( \frac{\Gamma_e}{\delta x_{PE}} A_e + \frac{\Gamma_w}{\delta x_{WP}} A_w - S_p \right) \phi_P = \left( \frac{\Gamma_w}{\delta x_{WP}} A_w \right) \phi_W + \left( \frac{\Gamma_e}{\delta x_{PE}} A_e \right) \phi_E + S_u \right]$$
(4.10)



$$\left[ \left( \frac{\Gamma_e}{\delta x_{PE}} A_e + \frac{\Gamma_w}{\delta x_{WP}} A_w - S_p \right) \phi_P = \left( \frac{\Gamma_w}{\delta x_{WP}} A_w \right) \phi_W + \left( \frac{\Gamma_e}{\delta x_{PE}} A_e \right) \phi_E + S_u \right]$$
(4.10)

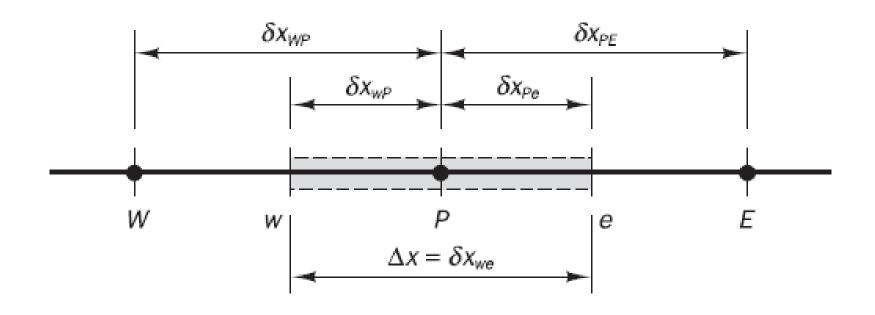


$$\left(\frac{\Gamma_e}{\delta x_{PE}} A_e + \frac{\Gamma_w}{\delta x_{WP}} A_w - S_p\right) \phi_P = \left(\frac{\Gamma_w}{\delta x_{WP}} A_w\right) \phi_W + \left(\frac{\Gamma_e}{\delta x_{PE}} A_e\right) \phi_E + S_u \tag{4.10}$$

$$a_P \phi_P = a_W \phi_W + a_E \phi_E + S_u$$

$$a_P \phi_P = a_W \phi_W + a_E \phi_E + S_u$$

$a_W$	$a_E$	$a_P$
$\frac{\Gamma_{w}}{\delta x_{WP}} A_{w}$	$\frac{\Gamma_e}{\delta x_{PE}} A_e$	$a_W + a_E - S_P$



- Step III: Solution of Equation
  - Direct Method
  - Iterative Method

In Chapter 7 we describe matrix solution methods that are specially designed for CFD procedures. The techniques of dealing with different types of boundary conditions will be examined in detail in Chapter 9.

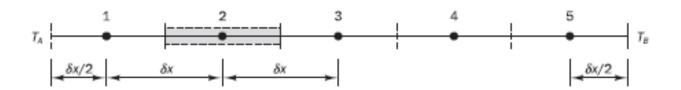
Example I: Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of 100°C and 500°C respectively. The one- dimensional problem sketched in Figure is governed by:

$$\frac{\mathrm{d}}{\mathrm{d}x} \left( k \frac{\mathrm{d}T}{\mathrm{d}x} \right) = 0$$

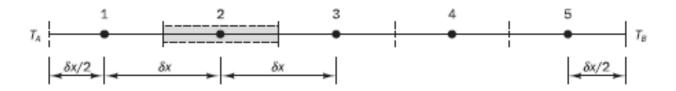
$$T_A = 100$$
Area (A)
$$T_B = 500$$

Calculate the steady state temperature distribution in the rod. Thermal conductivity k equals 1000 W/m.K, cross-sectional area A is  $10 \times 10^{-3}$  m<sup>2</sup>.

• Let us divide the length of the rod into five equal control volumes as shown in Figure ( $\delta x = 0.1 \text{ m}$ ).



Let us divide the length of the rod into five equal control volumes as shown in Figure ( $\delta x = 0.1 \text{ m}$ ).



• For each one of nodes 2, 3 and 4 temperature values to the east and west are available as nodal values.

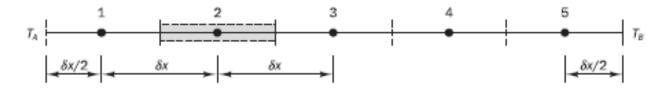
$$\left(\frac{k_{\varepsilon}}{\delta x_{PE}}A_{\varepsilon} + \frac{k_{w}}{\delta x_{WP}}A_{w}\right)T_{P} = \left(\frac{k_{w}}{\delta x_{WP}}A_{w}\right)T_{W} + \left(\frac{k_{\varepsilon}}{\delta x_{PE}}A_{\varepsilon}\right)T_{E}$$

The thermal conductivity  $(k_e = k_w = k)$ , node spacing  $(\delta x)$  and cross-sectional area  $(A_e = A_w = A)$  are constants. Therefore the discretised equation for nodal points 2, 3 and 4 is:

$$a_P T_P \!= a_W T_W \!+ a_E T_E$$

$a_W$	$a_E$	$a_P$
$\frac{k}{\delta x}A$	$\frac{k}{\delta x}A$	$a_W + a_E$

•  $S_u$  and  $S_p$  are zero in this case since there is no source term in the governing equation



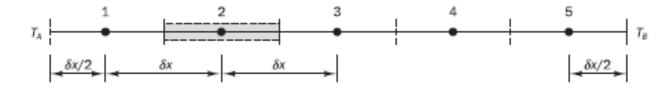
- Nodes 1 and 5 are boundary nodes, and therefore require special attention.
- For node 1:

$$\begin{split} kA \left( \frac{T_E - T_P}{\delta x} \right) - kA \left( \frac{T_P - T_A}{\delta x/2} \right) &= 0 \\ \left( \frac{k}{\delta x} A + \frac{2k}{\delta x} A \right) T_P &= 0 \; . \; T_W + \left( \frac{k}{\delta x} A \right) T_E + \left( \frac{2k}{\delta x} A \right) T_A \end{split}$$

• Discretised equation for boundary node 1:

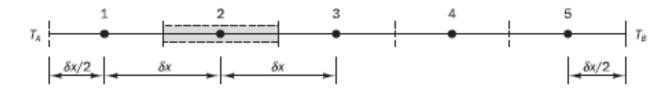
$$a_P T_P = a_W T_W + a_E T_E + S_u$$

$a_W$	$a_E$	$a_P$	$S_P$	$S_u$
0	$\frac{kA}{\delta x}$	$a_W + a_E - S_p$	$-\frac{2kA}{\delta x}$	$\frac{2kA}{\delta x}T_A$



Discretised equation for boundary node 5:

$$kA\left(\frac{T_B - T_P}{\delta x/2}\right) - kA\left(\frac{T_P - T_W}{\delta x}\right) = 0 \tag{4.19}$$



$$\left(\frac{k}{\delta x}A + \frac{2k}{\delta x}A\right)T_P = \left(\frac{k}{\delta x}A\right)T_W + 0 \cdot T_E + \left(\frac{2k}{\delta x}A\right)T_B \tag{4.20}$$

The discretised equation for boundary node 5 is

$$a_{P}T_{P} = a_{W}T_{W} + a_{E}T_{E} + S_{u}$$
(4.21)

where

$a_W$	$a_E$	$a_P$	$S_P$	$S_u$
$\frac{kA}{\delta x}$	0	$a_W + a_E - S_p$	$-\frac{2kA}{\delta x}$	$\frac{2kA}{\delta x}T_B$

$a_W$	$a_E$	$a_P$	$S_P$	$S_u$
0	$\frac{kA}{\delta x}$	$a_W + a_E - S_p$	$-\frac{2kA}{\delta x}$	$\frac{2kA}{\delta x}T_A$

$a_W$	$a_E$	$a_P$
$\frac{k}{\delta x}A$	$\frac{k}{\delta x}A$	$a_W + a_E$

$a_W$	$a_E$	$a_P$	$S_P$	$S_u$
$\frac{kA}{\delta x}$	0	$a_W + a_E - S_p$	$-\frac{2kA}{\delta x}$	$\frac{2kA}{\delta x}T_B$

$a_W$	$a_E$	$a_P$	$S_P$	$S_u$
0	$\frac{kA}{\delta x}$	$a_W + a_E - S_p$	$-\frac{2kA}{\delta x}$	$\frac{2kA}{\delta x}T_A$

$a_W$	$a_E$	$a_P$
$\frac{k}{\delta x}A$	$\frac{k}{\delta x}A$	$a_W + a_E$

$a_W$	$a_E$	$a_P$	$S_P$	$S_u$
$\frac{kA}{\delta x}$	0	$a_W + a_E - S_p$	$-\frac{2kA}{\delta x}$	$\frac{2kA}{\delta x}T_B$

• For  $kA/\delta x = 100$  is:

$$300T_1 = 100T_2 + 200T_A$$
  
 $200T_2 = 100T_1 + 100T_3$   
 $200T_3 = 100T_2 + 100T_4$   
 $200T_4 = 100T_3 + 100T_5$   
 $300T_5 = 100T_4 + 200T_B$ 

Node	$a_W$	$a_E$	$S_u$	$S_P$	$a_P = a_W + a_E - S_P$
1	0	100	$200T_A$	-200	300
2	100	100	0	0	200
3	100	100	0	0	200
4	100	100	0	0	200
5	100	0	$200T_B$	-200	300

Node	$a_W$	$a_E$	$S_u$	$S_P$	$a_P = a_W + a_E - S_P$
1	0	100	$200T_A$	-200	300
2	100	100	0	0	200
3	100	100	0	0	200
4	100	100	0	0	200
5	100	0	$200T_B$	-200	300

$$\begin{bmatrix} 300 & -100 & 0 & 0 & 0 \\ -100 & 200 & -100 & 0 & 0 \\ 0 & -100 & 200 & -100 & 0 \\ 0 & 0 & -100 & 200 & -100 \\ 0 & 0 & 0 & -100 & 300 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 200T_A \\ 0 \\ 0 \\ 200T_B \end{bmatrix}$$

• For  $T_A = 100$  and  $T_B = 500$  the solution of equation can obtained by using, for example, Gaussian elimination:

$$egin{array}{c|c} T_1 & 140 \ T_2 & 220 \ T_3 & 300 \ T_4 & 380 \ T_5 & 460 \ \end{array}$$

• For  $T_A = 100$  and  $T_B = 500$  the solution of equation can obtained by using, for example, Gaussian elimination:

